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A q Theory of Internal Capital Markets

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ABSTRACT

We propose a tractable model of dynamic investment, spinoffs, financing, and risk management for a multidivision firm facing costly external finance. Our analysis formalizes the following insights: (i) Within-firm resource allocation is based not only on divisions' productivity, as in winner-picking models, but also their risk; (ii) firms may voluntarily spin off productive divisions to increase liquidity; (iii) diversification can reduce firm value in low-liquidity states, as it increases the spinoff cost and hampers liquidity management; (iv) corporate socialism makes liquidity less valuable; and (v) division investment is determined by the ratio between marginal q and marginal value of cash.

MULTIDIVISION FIRMS—THAT IS, FIRMS that operate two or more divisions and allocate resources to their divisions through an internal capital market—play an important role in the economy. For example, Maksimovic and Phillips (2002) estimate that multidivision firms account for about three-fourths of total output in the U.S. manufacturing sector.

The empirical literature shows that multidivision firms behave very differently compared to stand-alone firms. These differences are found across firm

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policies, including those at the very core of corporate finance—cash management, financing, and investment decisions. For example, multidivision firms tend to hold less cash (Duchin (2010)), are more resilient in the face of external capital market disruptions (Matvos and Seru (2014)), and actively reallocate resources across divisions (Giroud and Mueller (2015)). The objective of this paper is to propose a tractable dynamic framework that sheds light on the mechanics of multidivision firms, taking into account the complex and intertwined nature of their risk management, financing, and investment decisions.

Broadly speaking, the theoretical literature on multidivision firms can be classified into two camps, namely, the "bright side" and the "dark side" theories of internal capital markets. The bright side theories highlight the winner-picking role of headquarters (Alchian (1969), Williamson (1975), Stein (1997)). In these models, headquarters can create value by reallocating resources from less productive divisions toward more productive divisions (the "winners"). In contrast, dark side theories argue that internal capital markets are plagued with agency conflicts, as they give rise to internal politics in the allocation of resources. This view was first proposed by Coase (1937), who argued that power within a hierarchy impacts internal policies, and later formalized in the models of influence activities (Milgrom (1988), Milgrom and Roberts (1988), Meyer, Milgrom, and Roberts (1992)). In these models, managers of weaker divisions have an incentive to lobby headquarters for more resources, in an attempt to distort the resource allocation in their favor. To mitigate such inefficient lobbying, headquarters may find it optimal to tilt the resource allocation toward "corporate socialism" whereby stronger divisions cross-subsidize weaker ones (Rajan, Servaes, and Zingales (2000), Scharfstein and Stein (2000)).

While these models have been influential, they are subject to two main limitations. First, they typically take other policies (e.g., cash management) as given, and hence do not account for the interdependence across these policies. As we show, allowing for the joint determination of such policies often reverses the predictions from simpler models featuring fewer policies. Second, these models are static, and hence do not account for the changing conditions companies face in a dynamic environment. These limitations are nontrivial. In a dynamic world, firms can run low on cash, which generates a need for state-contingent and time-varying risk management policy. In turn, this can affect the way internal capital markets operate. For example, the notion of winner picking mentioned above, albeit well established in the literature, may need to be qualified. When companies run low on cash, the shareholder-value–maximizing policy may no longer be to allocate resources to high-productivity divisions, but instead to low-risk divisions. Or companies may decide to spin off entire divisions, preferring higher corporate cash holdings over diversification benefits (from retaining more divisions). More broadly, as these examples illustrate, it is important to consider the dynamic and intertwined nature of multidivision firms' policies when formulating a theory of internal capital markets.

This paper aims to fill this gap by providing a tractable dynamic framework in which cash management, external financing, dividend payout, division sale (spinoff), and investment (including cross-divisional transfers) are characterized simultaneously for a multidivision firm that faces costly external finance. Our framework builds on the model of Bolton, Chen, and Wang (2011), (henceforth BCW), for stand-alone firms. Compared to BCW, our framework has two main innovations. First, we consider a firm with two divisions. As such, our model has two key state variables: (i) the ratio of capital stock between the two divisions, denoted by z, which is new in our model; and (ii) the ratio between the liquid asset (cash) and the illiquid productive capital stock (the sum of capital stock in the two divisions), denoted by w. Second, we allow for lobbying frictions at the division level, in the spirit of the dark side models of internal capital markets. As we will show, our parsimonious framework captures many situations that multidivision firms face in practice and yields a rich set of prescriptions.

Our analysis formalizes the following insights. First, starting with the case without corporate socialism, we find that multidivision firms hold less cash, require lower amounts of external financing, and can more easily pay dividends compared to stand-alone firms. These predictions are intuitive—diversification decreases the volatility of the firm's cash flows and hence reduces the need for liquidity. This lower need for liquidity is consistent with Duchin's (2010) finding that multidivision firms tend to hold less cash than stand-alone firms.

Second, when firms run out of cash, they may optimally choose to spin off one of their divisions. Given the lumpy nature of division sales, the spinoff can generate more cash than what the firm needs to efficiently operate the remaining division, in which case the excess amount is paid out to shareholders as a special dividend. This result is consistent with Dittmar's (2004) finding that firms often pay a special dividend subsequent to a spinoff.[1] Another implication of the model is that diversification can make future division sales (when the firm runs low on cash and has to increase cash holdings via division sale) more costly. Taking both dimensions into account, our model implies both a dark and a bright side of diversification from the perspective of liquidity management, even for a shareholder-value–maximizing conglomerate. When liquidity is abundant, diversification reduces the need for liquidity. When liquidity is scarce, diversification can hamper the firm's liquidity management by making division sales less attractive as a way to replenish the firm's liquidity.

Third, we find that, when companies are flush with cash, they allocate more of their resources to the high-productivity division, as predicted by static models of winner picking. However, when cash is scarce, the risk management motive dominates and companies allocate more of their resources to the low-risk division. Taking both dimensions into account motivates a broader formulation of the winner-picking role of internal capital markets: When headquarters allocates resources to divisions, it does so based not only on productivity, but also on risk. In this regard, the within-firm allocation of resources is analogous to

[1] This result also speaks to the literature on leveraged buyouts (LBOs) that finds that, following LBO deals, LBO investors often sell entire divisions and subsequently pay out large dividends (Eckbo and Thorburn (2008)). While this is often seen as a form of asset stripping in the interest of LBO investors, our framework offers a potential shareholder-value–maximizing interpretation.

a dynamic portfolio choice problem in which funding is allocated based on the risk-return profile of the individual securities. Unlike the standard portfolio choice problem (e.g., Merton (1971)), the risk-neutral firm in our setting is endogenously risk averse. As we will show, this endogenous risk aversion depends not only on the firm's scaled cash balance, w, but also on the ratio of capital stock between the two divisions, z, as well as the cost of external financing and the cost of liquidating a division.

This insight has important implications for capital budgeting. Contrary to the textbook view, ignoring idiosyncratic risk and the balance sheet of the conglomerate when doing capital budgeting is incorrect—depending on the firm's liquidity, it may be optimal to invest in a lower net present value (NPV) project if the project's idiosyncratic risk is sufficiently low. Introducing a project (division) changes the firm's entire balance sheet composition and risk profile. As such, the firm should value the new project by computing the net value difference caused by introducing the new project into the firm, as opposed to evaluating the project as if it were a stand-alone project.

Fourth, we find that corporate socialism reduces the value of the firm, exacerbates underinvestment, and hampers the winner picking. While these results are intuitive, one subtle implication of socialism is that division sales become less costly with socialism than without, as spinning off a division (and becoming a stand-alone firm) eliminates socialism frictions and hence is more valuable for a conglomerate with socialism. This has implications for liquidity management. Indeed, with socialism, liquidity is less valuable since it is less costly to replenish the firm's liquidity through a spinoff.

Fifth, our model offers insights on the q theory of investment for a conglomerate. In neoclassical settings in which the Modigliani-Miller (MM) theorem holds, the value of a conglomerate is simply the sum of its divisions' values. In contrast, in our model, the conglomerate's division-level investment decisions depend on not only liquidity (as in BCW), but also the relative (capital stock) size of the two divisions. With convex adjustment costs, a financially constrained conglomerate equates the ratio between the marginal q and the marginal cost of investing in each division to the marginal value of cash.[2] This is a generalized version of the q theory of investment for a financially constrained single-division firm analyzed in BCW.

In addition, we provide several extensions of our baseline model. In particular, in one extension, we allow for capital redeployability across divisions, that is, we assume that conglomerates can redeploy physical capital at little cost from one division to another. We show that capital redeployability contributes to the bright side of internal capital markets, increasing the value of the conglomerate by enhancing the conglomerate's ability to channel resources toward the more productive division. In another extension, we generalize our model to

[2] The assumption of convex adjustment costs allows us to simplify the analysis by leaving out the possibility that inaction is optimal.

account for the initial transition of a single-division firm into a conglomerate, and we characterize how the endogenous formation of the conglomerate can give rise to either a conglomerate discount or premium.

Related Literature. Our paper is related to several strands of the literature. First, it is related to the few but notable studies that use dynamic modeling to study the behavior of multidivision firms. In particular, Gomes and Livdan (2004) use a dynamic model to examine the valuation implications of diversification.[3] Matvos and Seru (2014) estimate a structural dynamic model that quantifies the extent to which internal capital markets helped offset the financial market disruptions that occurred during the financial crisis of 2007 to 2010. Bakke and Gu (2017) examine the rationales as to why multidivision firms hold less cash than stand-alone firms, estimating a structural dynamic model that quantifies the respective importance of selection (when a stand-alone firm endogenously becomes a multidivision firm) and diversification. Compared to these articles, our paper focuses on the interconnections between the various policies of multidivision firms. This allows us to provide a rich set of prescriptions that speak to various aspects of internal capital markets, ranging from the validity of winner-picking predictions to the economics of spinoffs.

Second, our paper is related to the large literature that uses dynamic models of financially constrained firms to characterize their investment, financing, and risk management decisions (e.g., Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2007), Riddick and Whited (2009), Décamps et al. (2011), Bolton, Chen, and Wang (2013), Hugonnier, Malamud, and Morellec (2015), Nikolov, Schmid, and Steri (2019), Abel and Panageas (2020)).[4] Also related is the work of Malenko (2019), who uses a dynamic model to study the optimal capital budgeting mechanism in a single-division firm.

While our framework shares various features with models proposed in this literature, the key difference is our focus on a two-division firm as opposed to a representative single-division firm modeled in these papers. As a result, our model, while parsimonious, is inevitably a two-dimensional problem involving partial differential equations (PDEs). This is a key departure from almost

[3] In their model, stand-alone firms diversify only when they become relatively unproductive in their current activities. This relates to the earlier models by Matsusaka (2001) and Maksimovic and Phillips (2002), who predict that firms diversify into industries that match their organizational capabilities and managerial resources, respectively.

[4] In our model, dynamic state-contingent liquidity management is optimal because external equity issuance is costly. The idea that dynamic liquidity and risk management are often optimal in response to financial frictions is quite robust and holds in more general settings. For example, in dynamic contracting models in which financial frictions endogenously arise due to moral hazard, limited commitment, and inalienability of human capital, the agent's promised utility is closely linked to and implementable with liquid asset holdings (e.g., cash or undrawn credit) and state-contingent contracts. A partial list of contracting-based liquidity and risk management models includes DeMarzo and Sannikov (2006), Biais et al. (2007, 2010), DeMarzo and Fishman (2007), Jiang et al. (2022), and Rebelo, Wang, and Yang (2022).

all existing models in the literature, whose formulations can be simplified to one-dimensional problems whose solutions are characterized by ordinary differential equations (ODEs).[5] Despite the richness of our model, we offer a theoretical framework that remains analytically tractable and economically intuitive, provide proofs of the key results, and numerically solve the model with high accuracy.

Third, our paper is related to the large empirical literature that studies the mechanics of internal capital markets. This literature finds support for both the bright and the dark side views. In particular, the findings of Maksimovic and Phillips (2002), Guedj and Scharfstein (2004), and Giroud and Mueller (2015) indicate that companies allocate resources in a value-enhancing manner. Naturally, this need not imply that internal capital markets achieve the first-best allocations. Indeed, the shareholder-value–maximizing formulation of our model does not deliver first-best allocations. In this regard, Shin and Stulz (1998), Rajan, Servaes, and Zingales (2000), and Ozbas and Scharfstein (2010) find evidence for distortions that is consistent with the models of corporate socialism.[6] Overall, the empirical evidence suggests that our model, which combines the bright and the dark sides—that is, firms striving to allocate resources in a value-maximizing fashion while facing rent-seeking behavior of their division managers—might provide a realistic characterization of internal capital markets.[7]

Lastly, our paper is related to the literature on corporate spinoffs (e.g., Maksimovic and Phillips (2001), Dittmar (2004), Eckbo and Thorburn (2008)). In particular, and as we mention above, our predictions that firms tend to pay out a special dividend following a spinoff is consistent with the empirical findings of Dittmar (2004).

The remainder of this paper is organized as follows. Section I presents the baseline model. Section II characterizes the first-best solution. Section III extends the baseline model to allow for corporate socialism. Section IV presents the model solution. Sections V and VI provide a quantitative analysis without and with corporate socialism, respectively. Section VII extends the model to allow for capital redeployability across divisions. Section VIII presents several other extensions. Finally, Section IX concludes.

[5] Mathematically, we characterize the solution of a diversified firm's two-dimensional optimization problem by using a variational-inequality method and provide a verification theorem along with additional technical results. Our paper is among the first to provide a verification theorem proof for a control problem that combines convex control, singular control, impulse control, and optimal stopping. Bolton, Wang, and Yang (2019a) feature singular control, impulse control, and optimal stopping but not convex control.

[6] Direct evidence of influence activities at the division level is provided by Duchin and Sosyura (2013) and Glaser, Lopez-De-Silanes, and Sautner (2013). Relatedly, Graham, Harvey, and Puri (2015) provide survey evidence suggesting that the capital allocation is often based on the division managers' reputation.

[7] For a review of the empirical literature on internal capital markets, see Maksimovic and Phillips (2013).

I. Model

In the following, we introduce the diversified firm's production and investment technology, describe the firm's financing opportunities, and state the firm's optimization problem.

A. Firm and Division Technologies

A diversified firm has two divisions, a and b. Each division employs capital as its factor of production.[8] The price of capital is normalized to unity. We denote by K_t^s and I_t^s the level of capital stock and gross investment in division s at time t, respectively, where $s = a, b$. The capital stock K_t^s of division s evolves according to

$$dK_t^s = (I_t^s - \delta_s K_t^s)dt, \tag{1}$$

where δ_s is the constant depreciation rate of the capital stock of division s.

The operating revenue generated by division s is proportional to its capital stock K_t^s and is given by $K_t^s dA_t^s$, where dA_t^s is the productivity shock for division s over time interval $(t, t+dt)$. We assume that under the risk-neutral measure $\mathbb{Q}$ (i.e., on a risk-adjusted basis), the cumulative (undiscounted) productivity of division s, A_t^s, follows an arithmetic Brownian motion process,

$$dA_t^s = \mu_s dt + \sigma_s d\mathcal{Z}_t^s, \quad s = a, b, \tag{2}$$

where $\mathcal{Z}_t^s$ is a standard Brownian motion under $\mathbb{Q}$, and μ_s and σ_s denote the mean and volatility of the division's productivity for a unit of time under the risk-adjusted measure.[9] We denote by ρ the constant correlation coefficient between the productivity shocks of the two divisions. That is, the quadratic covariation between $\mathcal{Z}_t^a$ and $\mathcal{Z}_t^b$, $d[\mathcal{Z}^a, \mathcal{Z}^b]_t$, is equal to ρdt. Note that the firm's productivity process in our model is a two-division generalization of the one used in BCW.[10]

Let dY_t^s denote the operating profit generated by division $s = a, b$ over increment dt,

$$dY_t^s = K_t^s dA_t^s - I_t^s dt - G_t^s dt. \tag{3}$$

Three terms contribute to the change in the division's operating profit dY_t^s. The first term in (3) is the division's operating revenue, the second term is the

[8] Eberly and Wang (2010) develop a general equilibrium q-theory of investment with the same two-sector setting.

[9] By directly specifying the joint productivity process for the firm's divisions under the risk-neutral measure, we incorporate the effect of the risk premium on the firm's decisions and valuation.

[10] The same i.i.d. productivity assumption is also made in dynamic contracting models, for example, DeMarzo and Sannikov (2006), Biais et al. (2007), DeMarzo and Fishman (2007), DeMarzo et al. (2012), and Hugonnier, Malamud, and Morellec (2015) absent investment.

investment (capital acquisition) cost, and the last term describes the capital adjustment cost.[11]

As in the q theory of investment (Lucas and Prescott (1971), Hayashi (1982), Abel and Eberly (1994)), we assume that the capital adjustment cost depends on investment and capital stock. That is, the capital adjustment cost in division s takes the form $G_t^s = G^s(I_t^s, K_t^s)$.

For analytical tractability, we assume that the adjustment costs for both divisions are homogeneous of degree one in their divisional I and K, so that

$$G_t^s = G^s\left(I_t^s, K_t^s\right) = g_s(i_t^s)\cdot K_t^s, \tag{4}$$

where $i_t^s = I_t^s/K_t^s$ denotes the investment-to-capital ratio of division s at time t. (The firm engages in asset sales at the division level when investment i_t is negative.) We apply this homogeneity property, which was first proposed by Lucas and Prescott (1971) and Hayashi (1982) for corporate investment, to investment at the division level.[12] We make the standard intuitive assumptions that $g_s(i)$ is increasing, smooth, and convex in i, as in the literature on the q theory of investment. Additionally, $g_s(0) = 0$.

The firm may choose to liquidate one or more divisions over time.[13] As we will show, it is suboptimal to liquidate both divisions at the same time. After liquidating division s, the firm receives liquidation value L_t^s and continues to operate as a going concern with the remaining division. Note that which division to liquidate at what time is endogenous. To preserve our model's homogeneity property, we assume that

$$L_t^s = \ell_s K_t^s, \tag{5}$$

[11] We can interpret this linear production function $K_t^s dA_t^s$ as an optimized outcome in a setting with constant returns to scale involving not only capital but also other flexibly adjustable factors of production. For instance, suppose the firm has a Cobb-Douglas production function with capital and labor, where both productivity $\{\vartheta_{t+1}\}$ and labor wage $\{\varepsilon_{t+1}\}$ shocks are i.i.d. In an MM world with perfect capital markets, for a given amount of capital K_t at any time t, it is optimal for the firm to solve the following static problem as labor N_t is fully and instantaneously adjustable: $\max_{N_t} \mathbb{E}_t(\vartheta_{t+1}K_t^{\alpha}N_t^{1-\alpha} - \varepsilon_{t+1}N_t)$. This yields the labor demand function $N_t^* = K_t((1-\alpha)\mathbb{E}_t(\vartheta_{t+1})/\mathbb{E}_t(\varepsilon_{t+1}))^{1/\alpha}$, which is proportional to capital K_t. We then obtain the realized revenue net of labor cost $o(\vartheta_{t+1}, \varepsilon_{t+1})K_t$, where $o(\vartheta_{t+1}, \varepsilon_{t+1}) = \left(\frac{\mathbb{E}_t(\varepsilon_{t+1})}{(1-\alpha)\mathbb{E}_t(\vartheta_{t+1})}\vartheta_{t+1} - \varepsilon_{t+1}\right)\left(\frac{(1-\alpha)\mathbb{E}_t(\vartheta_{t+1})}{\mathbb{E}_t(\varepsilon_{t+1})}\right)^{1/\alpha}$. The productivity shock dA_t in our continuous-time model then corresponds to $o(\vartheta_{t+1}, \varepsilon_{t+1})$ in the discrete-time formulation.

[12] Eberly, Rebelo, and Vincent (2009) provide empirical evidence in support of the Hayashi homogeneity assumption for the upper-size quartile of Compustat firms. For financial contracting models based on limited commitment (or inalienable human capital), it is sometimes more convenient to work with permanent shocks to capital (Ai and Li (2015), Bolton, Wang, and Yang (2019b)).

[13] To simplify the analysis, we assume that the (marginal) strategic buyer of the sold division is financially unconstrained (i.e., deep pocketed). Accordingly, the buyer only pays for the liquidated division's capital stock K^s at the ongoing market price. (An analogous setting is the fire-sale model of Shleifer and Vishny (1992).) Because the financially unconstrained buyer values cash at its face value (i.e., the buyer's marginal value of cash is one), the financially constrained conglomerate optimally retains all of its cash holdings inside the firm after selling the division. In Section VIII.A, we consider an alternative specification of the conglomerate's liquidation technology, in which the conglomerate optimally allocates a fraction of its cash holdings to the sold division.

where $\ell_s > 0$. The lower the value of ℓ_s, the more inefficient the liquidation technology for division s. Of course, the firm may eventually die as it may be optimal to also liquidate the remaining division in the future.

To focus on the economically interesting case, we impose the following conditions:

$$\mu_a > \ell_a \cdot (r + \delta_a) \quad \text{and} \quad \mu_b > \ell_b \cdot (r + \delta_b). \tag{6}$$

Otherwise, the firm prefers immediate liquidation without using its production technology.

The firm's operating cash flow, dY_t, over time increment dt is given by

$$dY_t = dY_t^a + dY_t^b = \left(K_t^a dA_t^a + K_t^b dA_t^b\right) - \left(I_t^a + I_t^b + G_t^a + G_t^b\right)dt. \tag{7}$$

Let τ_L denote the firm's (stochastic) liquidation/death time. If $\tau_L = \infty$, the firm never dies but may operate with only one division. For a single-division firm, $\tau_L = \tau_s$, as there is only one division s. However, for a two-division firm, division sale and firm liquidation are very different events. We thus differentiate between three stopping times: τ_a, τ_b, and τ_L.

B. External Financing Costs and Cash Management

Neoclassical investment models (Hayashi (1982)) assume that the firm faces frictionless capital markets and that the MM theorem holds. In reality, however, firms often face external financing costs due to various financial frictions, for example, transaction costs, asymmetric information, and managerial incentive problems.[14]

B.1. External Financing Costs

We do not explicitly model the microfoundations of financing costs. Instead, we directly specify equity issuance costs as in the literature. Specifically, as in BCW, we assume that a firm incurs both a fixed cost Φ and a proportional (marginal) cost γ whenever it chooses to issue external equity. Together, these costs imply that the firm will optimally tap equity markets only intermittently, and when doing so it raises funds in lumps, consistent with observed firm behavior.

To preserve our model's homogeneity property, we assume that the firm's fixed cost of issuing equity at t is proportional to its total capital stock K_t. That is, the fixed cost of equity issuance, Φ_t, is given by

$$\Phi_t = \phi K_t = \phi \cdot \left(K_t^a + K_t^b\right), \tag{8}$$

where $\phi > 0$ is a constant measuring the fixed equity issuance cost.

[14] The classic writings include Jensen and Meckling (1976), Leland and Pyle (1977), and Myers and Majluf (1984).

In practice, external costs of financing scaled by firm size may decrease with firm size. With this caveat in mind, we point out that there are conceptual, mathematical, and economic reasons for modeling these costs as proportional to firm size. First, by modeling the fixed financing costs proportional to firm size, we ensure that the firm does not grow out of the fixed costs.[15] Second, the information and incentive costs of external financing may to some extent be proportional to firm size. Indeed, the negative announcement effect of a new equity issue affects the firm's entire capitalization. Similarly, the negative incentive effect of a more diluted ownership may have costs that are proportional to firm size. Finally, this assumption keeps the model tractable and generates stationary dynamics for the firm's cash-to-capital ratio.[16]

We denote by H_t the firm's cumulative external financing up to time t with $H_0 = 0$ and by dH_t the firm's incremental external financing over time interval $(t, t + dt)$. Similarly, let X_t denote the cumulative costs of external financing up to time t with $X_0 = 0$, and dX_t the incremental costs of raising incremental external funds dH_t. The cumulative external equity issuance H and the associated cumulative costs X are stochastic controls chosen by the firm.

Technically, due to the fixed equity issuance costs, the firm's external financing policy can be described as a tuple $\nu = \{\tau^{(1)}, \tau^{(2)}, \ldots; M^{(1)}, M^{(2)}, \ldots\}$, where $\tau^{(i)}$ represents the i^{th} external financing (stopping) time, and $M^{(i)} > 0$ represents the corresponding net financing amount at the i^{th} financing time. When the firm issues no equity, that is, $t \neq \tau^{(i)}$, we have $dH_t = dX_t = 0$. When the firm issues equity, that is, $t = \tau^{(i)}$, we have

$$H_{\tau^{(i)}} = H_{\tau^{(i)}-} + M^{(i)}, \tag{9}$$

$$X_{\tau^{(i)}} = X_{\tau^{(i)}-} + \Phi_{\tau^{(i)}} + \gamma M^{(i)}. \tag{10}$$

Equations (9) and (10) imply that the net equity raised is $dH_t = M^{(i)}$ and the cost of financing is $dX_t = \Phi_{\tau^{(i)}} + \gamma M^{(i)}$ at $t = \tau^{(i)}$. Here, $\tau^{(i)}-$ refers to the time immediately before $\tau^{(i)}$.

B.2. Cash Carry Costs and Cash Management

Let W_t denote the firm's cash balance at t. If the firm's cash is positive, it survives with probability one. However, if the firm runs out of cash ($W_t = 0$), it has to either raise external funds to continue operating or liquidate one of its divisions to replenish cash.

[15] Indeed, this is a common assumption in the investment literature. See Cooper and Haltiwanger (2006) and Riddick and Whited (2009), among others. If the fixed cost is independent of firm size, it will not matter when firms become sufficiently large in the long run.

[16] A potential limitation of our model is that it will be misspecified as a structural model of firms' outside equity issue decisions. As such, the model is likely to work best when applied to mature firms as opposed to start-ups and small entrepreneurial firms. Nevertheless, this limitation is mitigated in our setting, since conglomerates (or, more generally, multidivision firms) tend to fit the former category.

If the firm chooses to raise external funds, it incurs both the fixed and marginal financing costs specified above. In some situations, the firm may prefer selling one of its divisions even before exhausting its cash balance. As we discuss in detail below, this result is a novel insight from our multidivision firm model. In contrast, it is not optimal for a single-division firm to liquidate itself as long as it still has a positive cash balance (BCW).

As in most cash management models, the rate of return that the firm earns on its cash balance is the risk-free rate r minus a carry cost $\lambda > 0$ that captures in a simple way the agency costs that may be associated with free cash inside the firm.[17] However, paying out cash also reduces the firm's cash balance, which potentially exposes the firm to current and future underinvestment, and future external financing costs. This trade-off, which has been widely analyzed in the literature, determines the optimal payout policy. We denote by U_t the firm's cumulative (nondecreasing) payout to shareholders up to time t, and by dU_t the incremental payout over time interval dt. Distributing cash to shareholders may take the form of a special dividend or a share repurchase.

Combining cash flows from operations dY_t given in (7) with the firm's financing policy given by the cumulative payout process U and the cumulative external financing process H, in the region in which the firm neither sells a division nor liquidates, its cash balance W evolves according to

$$dW_t = dY_t + (r - \lambda)W_t dt + dH_t - dU_t, \tag{11}$$

where the second term is the interest income (net of the carry cost λ), the third term dH_t is the cash inflow from external financing, and the last term dU_t is the cash outflow to investors, so that $(dH_t - dU_t)$ is the net cash flow from financing. As equity issuance is costly, it is not optimal to simultaneously issue equity and pay out a dividend. That is, at all t, either $dH_t = 0$ or $dU_t = 0$. As raising external financing is costly, the firm is often financially constrained—it neither issues equity nor pays out a dividend ($dH_t = dU_t = 0$), even though saving inside the firm is also costly ($\lambda > 0$).

C. Firm Optimization

C.1. Single-Division Firm Optimization

Next, we state the optimization problem for a single-division firm, proposed by BCW, which serves as an important benchmark for at least two reasons. First, it allows us to characterize how having more than one division changes a firm's decisions and valuation. Second, as a multidivision firm may sell one or more of its divisions, the solution for a single-division firm naturally enters into our analysis of the optimization problem for a multidivision firm.

Let $P^s(K^s, W)$ denote the value of a single-division firm with division s, and let $\{K_t^s; t \geq 0\}$ be the firm's capital stock process and $\{W_t; t \geq 0\}$ its cash balance

[17] Alternatively, the cost of carrying cash may arise from tax distortions (e.g., Graham (2000)).

process. The firm chooses its investment I^s, payout policy U^s, external financing policy H^s, and liquidation time $\tau_L = \tau_s$ to maximize shareholder value by solving

$$\sup \mathbb{E}\left[\int_0^{\tau_s} e^{-rt}\left(dU_t^s - dH_t^s - dX_t^s\right) + e^{-r\tau_s}\left(L_{\tau_s}^s + W_{\tau_s}\right)\right]. \tag{12}$$

The expectation takes risk into account (i.e., under the risk-neutral measure $\mathbb{Q}$). The first term is the discounted value of the net payouts to shareholders, and the second term is the discounted value from liquidation. The firm may never liquidate (i.e., $\tau_s = \infty$).

C.2. Multidivision Firm Optimization

Unlike a single-division firm, which ceases to exist upon liquidating its only division, a multidivision firm can sell one or more divisions to replenish its cash balance, and continue operating as a going concern with the remaining divisions.

After selling a division, the conglomerate becomes a single-division firm that behaves as in BCW. Consider the case in which the conglomerate sells division b at time τ_b. The firm's cash balance then increases from W_{τ_b-} by a discrete amount $L^b_{\tau_b-}$ to the post–division-sale cash balance of

$$W^a_{\tau_b} = W_{\tau_b-} + L^b_{\tau_b-} = W_{\tau_b-} + \ell_b K^b_{\tau_b-}. \tag{13}$$

Similarly, after selling division a, the firm becomes a single-division firm with cash balance $W^b_{\tau_a} = W_{\tau_a-} + L^a_{\tau_a-} = W_{\tau_a-} + \ell_a K^a_{\tau_a-}$.

Let $F(K^a, K^b, W)$ denote the conglomerate's shareholder value. In Section IV.C, we show that it is never optimal for the firm to simultaneously sell both divisions (see Proposition 1). This is because the option value of keeping at least one division alive is strictly positive. We can therefore divide the conglomerate's optimization problem into two subproblems: one after it sells one of its divisions at stochastic time τ_s, and the other before the sale of the division. We solve the problem via backward induction.

Shareholders choose investment levels (I^a, I^b), division sale timing (τ_a, τ_b), payout policy U, and external financing H to maximize the conglomerate's value by solving

$$\sup \mathbb{E}\left[\int_0^{\tau} e^{-rt}(dU_t - dH_t - dX_t) + e^{-r\tau}\left\{P^a\left(K^a_\tau, W^a_\tau\right)1_{\{\tau=\tau_b\}} + P^b\left(K^b_\tau, W^b_\tau\right)1_{\{\tau=\tau_a\}}\right\}\right], \tag{14}$$

where $1_{\{\cdot\}}$ is the indicator function and $P^s(K^s_\tau, W^s_\tau)$ is the value of the single-division firm (with division s being the surviving one) defined in equation (12). The conglomerate spins off a division at stopping time τ given by $\tau = \min\{\tau_a, \tau_b\}$ and liquidates itself at $\tau_L = \max\{\tau_a, \tau_b\}$.

In sum, the firm's optimization problem is a combined convex control (investment), singular control (payout), impulse control (equity issuance), and optimal stopping (division sale) problem (see Section I of the Internet Appendix).[18]

Finally, the conglomerate's average q, which is the ratio of its enterprise value $F(K_t^a, K_t^b, W_t) - W_t$ and its total capital stock $K_t^a + K_t^b$, is given by

$$q_t = \frac{F(K_t^a, K_t^b, W_t) - W_t}{K_t^a + K_t^b}. \tag{15}$$

Since our model is homogeneous of degree one in (K^a, K^b, W), as we will show below, we can write the average q as

$$q_t = q(z_t, w_t), \tag{16}$$

where z_t is the relative size of division a, given by the ratio of K_t^a and the total capital stock,

$$z_t = \frac{K_t^a}{K_t^a + K_t^b}, \tag{17}$$

and w_t is the conglomerate's scaled cash holding,

$$w_t = \frac{W_t}{K_t^a + K_t^b}. \tag{18}$$

II. An MM First-Best Benchmark

Before solving our model for a financially constrained conglomerate, it is helpful to consider the special case in which equity issuance is costless. In this case, both the MM and Coase theorems hold. Whether the two divisions are organized as units within a conglomerate or as two separate firms makes no economic difference. Under either organizational structure, the first-best outcome is achievable as it is optimal for each division to choose its own first-best investment policy and financing is irrelevant.[19]

Each division operates the same technology as in Hayashi (1982). Therefore, the average q for division s is equal to its marginal q and satisfies the present-value relation

$$q_{FB}^s = \sup_{i^s} \frac{\mu_s - i^s - g_s(i^s)}{r + \delta_s - i^s}. \tag{19}$$

[18] The Internet Appendix is available in the online version of this paper on *The Journal of Finance* website.

[19] Our MM benchmark model is related to Crouzet and Eberly (2023), who develop a q-theory of investment with multiple capital stocks, but importantly with no financial frictions, to study the role of rents and intangibles for valuation.

The first-order condition (FOC) for investment also implies that i^s_{FB} satisfies

$$1 + g_s'(i^s_{FB}) = q^s_{FB}, \tag{20}$$

which states that the division's marginal cost of investing is equal to the marginal benefit of investing, q^s_{FB}. Since adjustment costs are convex, (20) implies that the first-best investment is increasing in q. Note that q^s_{FB} is greater than unity, as the adjustment costs create a wedge between the value of installed capital and newly purchased capital.

The value of a conglomerate with cash W_t and divisional capital stocks K^a_t and K^b_t is given by

$$F^{FB}(K^a_t, K^b_t, W_t) = q^a_{FB}K^a_t + q^b_{FB}K^b_t + W_t. \tag{21}$$

The enterprise value of the conglomerate is equal to the value of the conglomerate minus cash, $F^{FB}(K^a_t, K^b_t, W_t) - W_t$, which is independent of W_t since MM holds.

Using the definition of average q given in equation (15), we obtain the following expression for the conglomerate's average q under the first-best, $q_{FB,t}$:

$$q_{FB,t} = z_t q^a_{FB} + (1 - z_t) q^b_{FB}. \tag{22}$$

That is, the average q of the conglomerate is simply a weighted average of the average q of its divisions, where the weights are the divisions' relative sizes, z_t and $(1 - z_t)$.

III. Corporate Socialism

While putting two divisions together as a firm provides diversification benefits, doing so may also give rise to agency costs. In particular, in the spirit of the models of influence activities (e.g., Milgrom (1988), Milgrom and Roberts (1988), Meyer, Milgrom, and Roberts (1992)), division managers may lobby headquarters to channel more of the firm's resources toward their division. In these models, division managers prefer larger resource allocations due to rent-seeking motives (e.g., if financial compensation, perquisite consumption, or outside job opportunities are linked to the size of the division they manage) or empire-building preferences (e.g., if managers enjoy the power and status of managing a larger division), and lobby headquarters accordingly. This lobbying incentive is especially pronounced for managers of weak divisions that face a higher risk of being downsized—their managers have incentives to overstate the division's true prospects in an attempt to access corporate resources that can be used to prevent or delay the downsizing.

Such lobbying activities are costly to the firm, as division managers devote time and effort lobbying headquarters at the expense of more productive activities. In the models of corporate socialism (e.g., Rajan, Servaes, and Zingales (2000), Scharfstein and Stein (2000), Matvos and Seru (2014)), headquarters can mitigate this lobbying behavior by tilting the resource allocation toward weaker divisions at the expense of stronger ones—analogous to a "socialist" outcome in which stronger divisions cross-subsidize weaker ones.[20]

In what follows, we consider two forms of socialism. In Section III.A, we consider the case in which socialism applies to firms' ongoing operations (i.e., division managers lobby headquarters for more resources to be channeled toward their division). In Section III.B, we consider the case in which socialism also applies to the spinoff decision (i.e., division managers also lobby against a potential spinoff of their division).

A. Socialism: Ongoing Operations Only

We model inefficient resource allocation within a firm by assuming that there is an additional cost that the firm pays by having two divisions inside the firm. Let G_t^c denote this cost, where c refers to the conglomerate. This cost can be interpreted as an influence cost, which lowers divisions' productivity and causes output losses.

To be more precise, the firm's operating profit, dY_t, over time increment dt is given by

$$dY_t = dY_t^a + dY_t^b - G_t^c dt = \left(K_t^a dA_t^a + K_t^b dA_t^b\right) - \left(I_t^a + I_t^b + G_t^a + G_t^b\right)dt - G_t^c dt. \tag{23}$$

We focus on socialism for the case in which the productivities of the two divisions are different. Without loss of generality, we refer to division a as the stronger division throughout the paper (i.e., $\mu_a \geq \mu_b$) whenever we study corporate socialism.

We model the cost of corporate socialism by using the following adjustment cost function at the conglomerate level:

$$G_t^c = \epsilon_t^a \mu_a K_t^a - \epsilon_t^b \mu_b K_t^b, \tag{24}$$

where $\epsilon_t^a \geq 0$ and $\epsilon_t^b \geq 0$ are stochastic processes to be specified.[21]

[20] Several empirical studies find that multidivision firms tend to overinvest in divisions with low investment opportunities and underinvest in those with high investment opportunities (e.g., Shin and Stulz (1998), Rajan, Servaes, and Zingales (2000), Ozbas and Scharfstein (2010)), consistent with the models of corporate socialism.

[21] In (24), the socialism costs depend on the size of the divisions (K^a and K^b), but not on the conglomerate's cash balance (W). In principle, influence activities can also depend on W—the more cash is available, the more division managers may engage in internal politics to divert some of this cash to their own divisions. To capture this intuition, we can generalize the socialism cost function (24) as

$$G_t^c = \epsilon_t^a \mu_a K_t^a - \epsilon_t^b \mu_b K_t^b + \epsilon^c(\mu_a - \mu_b)W_t, \tag{25}$$

where $\epsilon^c > 0$ is a constant and the last term captures the cost of socialism associated with the conglomerate's cash balance. Note that the coefficient on the last term is proportional to the productivity wedge $\mu_a - \mu_b$, consistent with the notion that influence activities are more severe when the productivity wedge between the two divisions is larger. Importantly, the last term in this more general formulation of socialism costs provides one economic interpretation for the cash-carry cost

The rationale behind equation (24) is as follows. First, to capture the notion that socialism is inefficient, thereby reducing the value of the conglomerate, we require $G_t^c \geq 0$ at all t, so that the firm's free cash flow is lower with socialism than without. Second, to model the inefficient resource transfer from the more productive division to the less productive one, we express G_t^c as the difference between two terms: the first term $\epsilon_t^a \mu_a K_t^a \geq 0$ captures the loss (in cash flow terms) at the more productive division a, while the second term $\epsilon_t^b \mu_b K_t^b \geq 0$ captures the gain at the less productive division b due to influence activities. As a whole, the conglomerate incurs a net cost that is given by the difference between the two terms.

In addition, conditional on the inefficient resource allocation away from the more productive division a to the less productive division b, we assume that G_t^c is symmetric as a function of the relative size of the two divisions. Specifically, we choose $\epsilon_t^a = \epsilon(1 - z_t)$ and $\epsilon_t^b = \epsilon(z_t)$, where $\epsilon(\cdot)$ is a linear function,

$$\epsilon(x) = \theta_c x, \quad x \in [0, 1], \tag{26}$$

and $\theta_c \geq 0$ is a constant describing the severity of socialism. The case with $\theta_c = 0$ corresponds to our baseline model of Section I with no corporate socialism. The higher the value of θ_c, the stronger corporate socialism. With the specific functional form in (26), we obtain

$$G_t^c = \theta_c \frac{(\mu_a - \mu_b) K_t^a K_t^b}{K_t^a + K_t^b} = g_c(z_t)\Big(K_t^a + K_t^b\Big), \tag{27}$$

where $g_c(z)$ is the scaled socialism cost function:

$$g_c(z) = \theta_c(\mu_a - \mu_b) z(1 - z). \tag{28}$$

Note that, by construction, $G^c \geq 0$ and $g_c(z)$ are symmetric in z, the relative size of the two divisions, and are higher the more balanced the two divisions are. This conveys the intuition that internal politics is most pronounced when both divisions are equally powerful in terms of size (i.e., when they both account for 50% of the firm), while internal politics is less of a concern when one division is larger than the other (say, if one division accounts for 99% of the firm and the other only 1%). Moreover, the larger the productivity wedge between the two divisions $(\mu_a - \mu_b)$, the larger the socialism cost. This echoes the models of influence activities (Milgrom (1988), Milgrom and Roberts (1988), Meyer, Milgrom, and Roberts (1992)), in which the less productive division has stronger incentives to engage in internal politics. Finally, we note that, in the above formulation, the socialism cost G_t^c can be interpreted as a tax on capital, where $g_c(z_t)$ represents the "effective" tax rate.

To see how socialism in our model makes the more productive division a less productive and inefficiently subsidize the less productive division b, we rewrite

(λ) that we take as exogenously given in our baseline model. To be precise, λ in our baseline model corresponds to $\epsilon^c(\mu_a - \mu_b)$ in (25).

the firm's operating profits in (23) as

$$dY_t = K_t^a\left(\widehat{\mu}_t^a dt + \sigma^a dZ_t^a\right) + K_t^b\left(\widehat{\mu}_t^b dt + \sigma^b dZ_t^b\right) - \left(I_t^a + I_t^b + G_t^a + G_t^b\right)dt, \quad (29)$$

where $\widehat{\mu}_t^a = \widehat{\mu}^a(z_t)$ and $\widehat{\mu}_t^b = \widehat{\mu}^b(z_t)$ can be interpreted as "compromised" productivities due to socialism for the two divisions,

$$\widehat{\mu}^a(z_t) = \mu_a(1 - \theta_c(1 - z_t)) \leq \mu_a, \quad (30)$$

$$\widehat{\mu}^b(z_t) = \mu^b(1 + \theta_c z_t) \geq \mu_b. \quad (31)$$

Equations (30) and (31) convey the idea that corporate socialism effectively reduces the productivity of the more productive division a to $\widehat{\mu}_t^a$, but increases the productivity of the less productive division b to $\widehat{\mu}_t^b$ from the headquarters' perspective. Naturally, the outcome is socially inefficient as the productivity loss at division a outweighs the productivity gain at division b, thereby reducing shareholder value.

In sum, headquarters chooses investment levels $(\widehat{I}^a, \widehat{I}^b)$, division sale timing $(\widehat{\tau}_a, \widehat{\tau}_b)$, payout $\widehat{U}$, and external financing $\widehat{H}$ to solve

$$\widehat{F}(K^a, K^b, W) = \sup_{\widehat{I}^a, \widehat{I}^b, \widehat{\tau}_a, \widehat{\tau}_b, \widehat{U}, \widehat{H}} \mathbb{E}\left[\int_0^{\widehat{\tau}} e^{-rt}(d\widehat{U}_t - d\widehat{H}_t - d\widehat{X}_t) + e^{-r\widehat{\tau}}\left\{P^a(K_{\widehat{\tau}}^a, W_{\widehat{\tau}}^a)1_{\{\widehat{\tau}=\widehat{\tau}_b\}} + P^b(K_{\widehat{\tau}}^b, W_{\widehat{\tau}}^b)1_{\{\widehat{\tau}=\widehat{\tau}_a\}}\right\}\right], \quad (32)$$

where $P^s(K_{\widehat{\tau}}^s, W_{\widehat{\tau}}^s)$ is the value of a single-division firm defined in equation (12).

Importantly, corporate socialism disappears if headquarters sells a division, as doing so eliminates the $G_t^c dt$ cost. As such, division sale is one way for the firm to mitigate corporate socialism. The parameters for the single-division firms' value functions $P^a(K_{\widehat{\tau}}^a, W_{\widehat{\tau}}^a)$ and $P^b(K_{\widehat{\tau}}^b, W_{\widehat{\tau}}^b)$ are the original (true) parameter values μ_a and μ_b. The conglomerate spins off a division at time $\widehat{\tau}$ given by $\widehat{\tau} = \min\{\widehat{\tau}_a, \widehat{\tau}_b\}$. The firm's liquidation time $\widehat{\tau}_L$ is then given by $\widehat{\tau}_L = \max\{\widehat{\tau}_a, \widehat{\tau}_b\}$. Naturally, firm value is lower with socialism than without:

$$\widehat{F}(K^a, K^b, W) \leq F(K^a, K^b, W; \mu_a, \mu_b). \quad (33)$$

B. Socialism: Both Ongoing Operations and Division Sale

Division managers' rent-seeking activities may not only distort the resource allocation on an ongoing basis, but also influence the headquarters' decision to spin off a division. Intuitively, when facing the risk of being spun off, division managers may engage in additional lobbying to fend off the spinoff. In what follows, we show that an inefficient conglomerate can persist longer due to the division manager's rent-seeking activities that inefficiently delay the headquarters' decision to divest the division.

We generalize our socialism model by assuming that the conglomerate incurs an additional cost of spinning off a division given by

$$G_t^d = \theta_d \Big[\mu^a (1 - z_t) \ell_a K_t^a - \mu^b z_t \ell_b K_t^b \Big] = g_d(z_t) \Big(K_t^a + K_t^b \Big) > 0, \tag{34}$$

where $\theta_d > 0$ measures the degree to which the division manager's rent-seeking activities influence the headquarters' spinoff decision, and $g_d(z)$ is a scaled cost function for spinoffs,

$$g_d(z) = \theta_d (\mu_a \ell_a - \mu_b \ell_b) z (1 - z) > 0. \tag{35}$$

Note that $G_t^d > 0$ follows from $\mu_a \ell_a > \mu_b \ell_b$, as division a is more productive ($\mu_a > \mu_b$) and the recovery value is higher ($\ell_a \geq \ell_b$).

The rationale for equations (34) and (35) is similar to that for equation (24). First, to capture the notion that socialism leads to inefficient value-destroying spinoff decisions, we require $G_t^d \geq 0$ at all t. Second, to model the inefficient resource transfer from the more productive division to the less productive one, we express G_t^d as the difference between two terms—the first term inside the square brackets $\mu^a (1 - z) \ell_a K_a \geq 0$ captures the loss (in value) at the more productive division a, while the second term $\mu^b z \ell_b K_b \geq 0$ captures the gain at the less productive division b due to influence activities. As a whole, the conglomerate incurs a net cost that is proportional to the difference between the two terms.

In analogy to the socialism cost function $g_c(z)$ for ongoing operations, $g_d(z)$ is also symmetric in z and is higher the more balanced the two divisions are. This conveys the intuition that internal politics is most pronounced when both divisions are equally powerful in terms of size. Moreover, the larger the wedge between the two divisions ($\mu_a \ell_a - \mu_b \ell_b$), the larger the socialism spinoff cost. Finally, we note that G_t^d can be interpreted as a one-time tax on capital for spinning off a division, where $g_d(z_t)$ represents the "effective" tax rate.

IV. Solution: Bright Side of Internal Capital Markets

In this section, we solve the model proposed in Section I. Firm value is a function of three state variables: the capital stock of each division, K^a and K^b, and the firm's cash balance, W. We solve the model by dividing the problem into three steps. First, we characterize the firm's decisions in the region in which the marginal source of financing is its internal financing. Second, we characterize the firm's optimal payout policies. Finally, we analyze how a financially constrained conglomerate dynamically replenishes its cash by choosing between external financing, division sale, and firm liquidation.

A. Interior Region

In this region, firm value $F(K^a, K^b, W)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} rF(K^a, K^b, W) = \sup_{I^a, I^b} \; & (I^a - \delta_a K^a) F_{K^a} + (I^b - \delta_b K^b) F_{K^b} \\ & + \left[(r-\lambda) W + \mu_a K^a + \mu_b K^b - (I^a + I^b + G^a + G^b) \right] F_W \\ & + \frac{1}{2} \left(\sigma_a^2 (K^a)^2 + \sigma_b^2 (K^b)^2 + 2\rho\sigma_a\sigma_b K^a K^b \right) F_{WW}. \end{aligned} \tag{36}$$

The first two terms (F_{K^a} and F_{K^b}) on the right-hand side of (36) capture the direct effects of investment on firm value, the third term (F_W) represents the effect of the firm's expected savings, and the last term (F_{WW}) captures the effect of the volatility of cash holdings W.

The firm finances its investment in both divisions out of the cash balance in this region. The divisional investment levels I^a and I^b satisfy the following interconnected FOCs[22]:

$$1 + G^a_{I^a}(I^a, K^a) = \frac{F_{K^a}(K^a, K^b, W)}{F_W(K^a, K^b, W)}, \tag{37}$$

$$1 + G^b_{I^b}(I^b, K^b) = \frac{F_{K^b}(K^a, K^b, W)}{F_W(K^a, K^b, W)}. \tag{38}$$

First, consider the special case with frictionless external and internal capital markets considered in Section II (i.e., the MM world). In this case, the marginal value of cash is $F_W = 1$, and the FOCs simplify to the neoclassical investment formula in (20)—that is, the firm's marginal q with respect to capital stock K^s in division s, $F_{K^s}(K^s, K^s, W)$, is equal to the firm's marginal cost of investing in division s, $1 + G^s_{I^s}$, and the two FOCs are independent of each other. In other words, one division's policy is independent of the other's policy (MM and Coase theorems).

These properties no longer hold in our setup with financing frictions. The left-hand side of (37) is the firm's marginal cost of increasing a unit of capital in division a, $1 + G^a_{I^a}$. The right-hand side is the marginal benefit, which is equal to the marginal q for division a, $F_{K^a}(K^a, K^b, W)$, divided by the marginal cost of financing (or equivalently, the marginal value of cash), $F_W(K^a, K^b, W)$. Optimality requires that the two sides of (37) be equal. The same reasoning applies to equation (38) with respect to division b. With costly external financing, the firm deploys cash optimally to both divisions so that the FOCs (37) and (38) for the two divisions hold and become interconnected.

[22] The convexity of the physical adjustment cost implies that the second-order condition is satisfied and the divisional investment decisions in our model admit interior solutions.

Rewriting (37) and (38), we have the FOCs

$$\frac{F_{K^a}(K^a, K^b, W)}{1 + G^a_{I^a}\left(I^a, K^a\right)} = \frac{F_{K^b}(K^a, K^b, W)}{1 + G^b_{I^b}\left(I^b, K^b\right)} = F_W(K^a, K^b, W). \tag{39}$$

The first equality states that the ratio between marginal q and the marginal cost of investing is equal for the two divisions—the implication of the intratemporal optimal allocation. The second equality describes the intertemporal optimal savings—the ratio between marginal q and the marginal cost of investing in all divisions is equal to the marginal value of savings (cash), $F_W(K^a, K^b, W)$. Note that while marginal q is well defined for each division, it is unclear how to define a meaningful marginal q at the conglomerate level, as $K^a + K^b$ is not a state variable; instead, both K^a and K^b are state variables.

By using the homogeneity property of our model and applying Euler's theorem, we obtain the expression

$$F(K^a, K^b, W) = F_{K^a}(K^a, K^b, W)K^a + F_{K^b}(K^a, K^b, W)K^b + F_W(K^a, K^b, W)W, \tag{40}$$

which links the book values of key balance sheet items (W, K^a, and K^b) to the firm's market value. Multiplying cash (W) and divisional capital stocks (K^a and K^b) by their respective marginal (shadow) values (i.e., the marginal value of cash F_W, the marginal q of division a's capital stock F_{K^a}, and the marginal q of division b's capital stock F_{K^b}) and then summing the three terms, we obtain the conglomerate's market value $F(K^a, K^b, W)$.

The homogeneity property also allows us to equivalently express the firm's three-state-variable value function as a two-state-variable value function,

$$F\left(K^a_t, K^b_t, W_t\right) = \left(K^a_t + K^b_t\right) \cdot f(z_t, w_t), \tag{41}$$

where z_t is given in (17) and w_t is the firm's cash-to-capital ratio defined in (18).

The ratio w between cash balance W_t and the firm's total physical capital stock $(K^a_t + K^b_t)$ is the key state variable measuring the firm's degree of financing constraints. Using Ito's Lemma, we obtain the following dynamics for w_t:

$$\begin{aligned} dw_t = {} & (r - \lambda)w_t dt + \left[z_t\left(\mu_a dt + \sigma_a d\mathcal{Z}^a_t\right) + (1 - z_t)\left(\mu_b dt + \sigma_b d\mathcal{Z}^b_t\right)\right] \\ & - \left[\left(i^a_t + g_a\left(i^a_t\right)\right)z_t + \left(i^b_t + g_b\left(i^b_t\right)\right)(1 - z_t)\right]dt \\ & - w_t\left[z_t\left(i^a_t - \delta_a\right) + (1 - z_t)\left(i^b_t - \delta_b\right)\right]dt. \end{aligned} \tag{42}$$

The first term reflects the firm's net interest income. The second term captures the operating revenues from the two divisions. The third term captures the total (flow) costs of investing. The last term captures the impact of changes in the divisions' capital stock.

In addition to w_t, the capital stock of division a as a fraction of the firm's total capital stock, $z_t = K_t^a/(K_t^a + K_t^b)$, measures the distribution of illiquid productive capital stocks between the two divisions, which is the other key state variable. Using the dynamics of K^a and K^b, we obtain the following process for $z_t \in [0, 1]$:

$$dz_t = z_t(1 - z_t)\Big[\big(i_t^a - \delta_a\big) - \big(i_t^b - \delta_b\big)\Big]dt. \tag{43}$$

The relative size of division a grows, that is, z_t increases, if and only if the growth rate of division a exceeds that of division b, that is, $(i_t^a - \delta_a) > (i_t^b - \delta_b)$.

By using our model's homogeneity property—for example, equation (41)—we can simplify the HJB equation (36) and obtain the following PDE for $f(z, w)$:

$$\mathcal{L}f(z, w) = 0, \tag{44}$$

where

$$\begin{aligned}\mathcal{L}f(z, w) = \sup_{i^a, i^b} \quad & (i^a - \delta_a)z\big[f(z, w) + (1 - z)f_z(z, w) - wf_w(z, w)\big] \\ & + (i^b - \delta_b)(1 - z)\big[f(z, w) - zf_z(z, w) - wf_w(z, w)\big] \\ & + \Big[(r - \lambda)w + (\mu_a - i^a - g_a(i^a))z + (\mu_b - i^b - g_b(i^b))(1 - z)\Big]f_w(z, w) \\ & + \frac{1}{2}\Big[\sigma_a^2 z^2 + \sigma_b^2(1 - z)^2 + 2z(1 - z)\rho\sigma_a\sigma_b\Big]f_{ww}(z, w) - rf(z, w).\end{aligned} \tag{45}$$

The first and second terms on the right-hand side of equation (45) capture the effects of investment in divisions a and b on firm value. The third term captures the effect of cash management. The fourth term captures the volatility effects (from both divisions and their correlation). The sum of these four terms represents the expected change in firm value. Subtracting $rf(z, w)$, the annuity value of $f(z, w)$, provides the net change in $f(z, w)$.

Note that it is optimal for the firm to rely solely on internal funds to finance both divisions' investments in this region. The conglomerate optimally chooses its divisional investments i^a and i^b so that the net change in its (scaled) value $f(z, w)$, $\mathcal{L}f(z, w)$, is zero.

The FOCs for divisional investment decisions can be simplified as

$$1 + g_a'(i^a) = \frac{f(z, w) + (1 - z)f_z(z, w)}{f_w(z, w)} - w, \tag{46}$$

$$1 + g_b'(i^b) = \frac{f(z, w) - zf_z(z, w)}{f_w(z, w)} - w. \tag{47}$$

The left-hand side of equation (46) is the marginal cost of investing in division a. The right-hand side is the marginal q of K^a divided by the marginal value of cash $f_w(z, w)$. The firm optimally chooses i^a by equating the two sides of (46). The same applies to division b in equation (47). As discussed above, the divisional investment decisions are interconnected because of financial constraints.

To fully characterize the value function $f(z, w)$, we must also analyze the firm's payout, external financing, and division sale decisions. We show that there are two other regions in the state space of (z, w): (i) a payout region in which the firm also actively pays out a dividend to shareholders and (ii) an external financing/division sale region in which the firm replenishes its cash by choosing external financing, division sale, or liquidation of the whole firm.

B. Payout Region

To determine the firm's payout region, we first consider the case in which the firm's cash holdings are very large relative to its size. In this case, the firm is better off paying out its excess cash to shareholders to avoid the cash-carrying costs. Let $\overline{w}_t$ denote the (stochastic) level of the cash-to-capital ratio w_t above which the firm pays out cash. As cum-dividend firm value $f(z_t, w_t)$ must be continuous at all t, the following value-matching condition holds:

$$f(z_t, w_t) = f(z_t, \overline{w}_t) + (w_t - \overline{w}_t), \quad \text{for } w_t > \overline{w}_t. \tag{48}$$

By taking the limit $w_t \to \overline{w}_t$ and calculating the derivative with respect to w_t, we obtain the following equation in the region in which $w_t \geq \overline{w}_t$:

$$f_w(z_t, w_t) = 1. \tag{49}$$

C. Division Sale, External Financing, and Liquidation Regions

When the firm is short on cash, it may raise costly external equity or liquidate a division to replenish its cash stock. An important difference from BCW is that a conglomerate may issue equity or liquidate a productive division to replenish its cash holdings before exhausting its cash. That is, the firm may move preemptively, as doing so alleviates even larger distortions in the future. Another key difference from BCW is that a conglomerate may choose equity issuance and division sale under different circumstances, while in BCW the firm either issues equity or liquidates itself when running out of cash. We show that the liquidation of the whole firm is the firm's last resort, as doing so wipes out its going-concern value (see Section II of the Internet Appendix).

PROPOSITION 1: *Under the conditions given in (6), (i) in the first-best world, it is never optimal to liquidate either division, while (ii) in a world with costly external financing, it is optimal for the firm to sequentially sell its divisions rather than liquidate the firm in its entirety.*

When the firm replenishes its cash via either equity issuance or division sale, it chooses the less costly option that yields a higher firm value. The choice depends on not only the liquidity ratio w_t, but also the relative size of the two divisions, captured by z_t, in addition to the structural parameters of the model.

C.1. Costly External Equity Issuance

First, we calculate firm value conditional on issuing external equity at t. Let $J(K_t^a, K_t^b, W_t)$ denote this conditional firm value given by

$$J\left(K_t^a, K_t^b, W_t\right) = \sup_{M_t>0} \quad F\left(K_t^a, K_t^b, W_t + M_t\right) - \left[\phi \cdot \left(K_t^a + K_t^b\right) + (1+\gamma)M_t\right]. \quad (50)$$

The first term on the right-hand side of equation (50) is the post–equity-issuance firm value, and the second term is the sum of net equity issuance M_t and the total cost of equity issuance, which includes the fixed equity issuance cost, $\phi \cdot (K_t^a + K_t^b)$, and the proportional issuance cost γM_t. Again, note that the value $J(K_t^a, K_t^b, W_t)$ is conditional on equity issuance, but equity issuance may not be optimal.

Let $\widetilde{F}(K_t^a, K_t^b, W_t)$ denote firm value conditional on external financing or division sale being optimal. That is, we have the following condition:

$$\widetilde{F}\left(K_t^a, K_t^b, W_t\right) = \max \left\{P^a\left(K_t^a, L_t^b + W_t\right),\ P^b\left(K_t^b, L_t^a + W_t\right),\ J\left(K_t^a, K_t^b, W_t\right)\right\}. \quad (51)$$

Equation (51) states that the firm selects one of the three mutually exclusive discrete choices to maximize its value. If sale of division a or b is optimal, firm value is given by the first and second terms, respectively, on the right-hand side of equation (51). If equity issuance is optimal, firm value is equal to $J(K_t^a, K_t^b, W_t)$ given in equation (50). The value function for the single-division firm is the same as in BCW and reported in Section I.

Let w_t^a denote the cash-to-capital ratio immediately after the conglomerate sells division b and becomes a stand-alone firm with only division a: $w_t^a = W_t^a / K_t^a$, where $W_t^a = L_t^b + W_t$. We define w_t^b analogously. Using the homogeneity property, we obtain

$$w_t^a = \frac{\ell_b(1-z_t) + w_t}{z_t} \quad \text{and} \quad w_t^b = \frac{\ell_a z_t + w_t}{1-z_t}. \quad (52)$$

Let m_t denote the scaled net equity issuance, $m_t = M_t/(K_t^a + K_t^b)$, and $j(z_t, w_t)$ denote firm value scaled by $(K_t^a + K_t^b)$, that is, $j(z_t, w_t) = J(K_t^a, K_t^b, W_t)/(K_t^a + K_t^b)$. Equation (50) can be simplified as

$$j(z_t, w_t) = \sup_{m_t>0} \ f(z_t, w_t + m_t) - \phi - (1+\gamma)m_t. \quad (53)$$

Intuitively, the marginal value of cash f_w must equal the marginal cost of financing $1+\gamma$ immediately after refinancing. This can be seen from the FOC $f_w = 1+\gamma$, implied by (53), provided that f is differentiable with respect to w.

Let $\widetilde{f}(z_t, w_t) = \widetilde{F}(K_t^a, K_t^b, W_t)/(K_t^a + K_t^b)$. Using the homogeneity property to simplify equation (51), we obtain

$$\widetilde{f}(z_t, w_t) = \max\left\{z_t\, p^a(w_t^a), (1-z_t)\, p^b\left(w_t^b\right), j(z_t, w_t)\right\}, \quad (54)$$

where w_t^a and w_t^b are given in equation (52).

The equation that defines the external financing/division sale regions is then given by

$$f(z_t, w_t) = \widetilde{f}(z_t, w_t). \tag{55}$$

In Section II.B of the Internet Appendix, we prove two technical results on refinancing and early liquidation.

D. Summary

In Section I of the Internet Appendix, we prove that the firm's (scaled) value $f(z, w)$ associated with the firm's optimal policies satisfies the variational inequality

$$\max\left\{\mathcal{L}f(z,w),\ 1 - f_w(z,w),\ \widetilde{f}(z,w) - f(z,w)\right\} = 0 \tag{56}$$

in the two-dimensional region defined by $z \in (0, 1)$ and $w \geq 0$.

Intuitively, the firm finds itself in one of the three regions. When the first term in equation (56) is equal to zero (and the other two terms are strictly negative), the firm is in the interior region and optimally chooses its investment-to-capital ratios for divisions a and b as prescribed by (46) and (47). When the second term is equal to zero, the conglomerate optimally makes a payout to shareholders as described by (48) and (49). Finally, when the last term is equal to zero, the firm optimally chooses either division sale or costly external financing, as captured by (53) and (54). We numerically solve the variational inequality in (56) by using a penalty-function–based iterative procedure described in Section III of the Internet Appendix.

V. Quantitative Analysis

We now turn to the quantitative analysis of the model. In Sections V.A, B, and C, we consider the case without socialism ($\theta_c = 0$) for a firm with two symmetric divisions, that is, two divisions whose productivity shocks have the same mean and volatility ($\mu_a = \mu_b$ and $\sigma_a = \sigma_b$) but are not perfectly correlated. [23] In Sections V.D, E, and F, we consider the case with asymmetric divisions. Finally, in Section V.G, we generalize our model to allow for debt financing.

Parameter choices. The parameters used in the benchmark case are provided in Table I. We set the annual mean and volatility of the productivity shocks to $\mu_a = \mu_b = 20\%$ and $\sigma_a = \sigma_b = 9\%$, respectively, which are in line with the estimates of Eberly, Rebelo, and Vincent (2009) for large U.S. firms.

While our analyses do not depend on the specific functional form of $g_s(i^s)$ for division $s = a, b$, for simplicity we adopt the following widely used quadratic

[23] In Section VI, we consider the case with socialism ($\theta_c > 0$) for a firm with asymmetric divisions.

Table I
Summary of Parameters

This table summarizes the symbols for the key parameters and values used in our quantitative analysis. The values in the "Symmetric" column are for the case in which the two divisions have the same parameter values. The values in the "Asymmetric" column are for the case in which the two divisions have different parameter values. Whenever applicable, parameter values are annualized.

Parameters	Symbol	Symmetric	Asymmetric
Risk-free rate	r	6%	6%
Proportional cash-carrying cost	λ	1%	1%
Proportional financing cost	γ	6%	6%
Fixed financing cost	ϕ	1%	1%
Correlation of two divisions	ρ	10%	10%
Risk-neutral mean productivity shock for division a	μ_a	20%	24%
Risk-neutral mean productivity shock for division b	μ_b	20%	16%
Volatility of productivity shock for division a	σ_a	9%	9%
Volatility of productivity shock for division b	σ_b	9%	9%
Rate of depreciation for division a	δ_a	9%	9%
Rate of depreciation for division b	δ_b	9%	9%
Adjustment cost parameter for division a	θ_a	8	8
Adjustment cost parameter for division b	θ_b	8	8
Socialism parameter	θ_c	0	1
Capital liquidation value for division a	ℓ_a/q^a_{FB}	0.6	0.6
Capital liquidation value for division b	ℓ_b/q^b_{FB}	0.6	0.6

form:

$$g_s(i^s) = \frac{\theta_s (i^s)^2}{2}, \tag{57}$$

where the parameter θ_s measures the degree of the adjustment cost for division s. For our baseline calculations, we assume that both divisions have the same adjustment cost parameter, which we set to $\theta_a = \theta_b = 8$ as in Shapiro (1986) and Hall (2001). We further assume that both divisions have the same annual depreciation rate, which we set to $\delta_a = \delta_b = 9\%$. Moreover, we assume that the liquidation value for a division is proportional to the market value (on a first-best basis) of the division's capital stock. This assumption captures the idea that, when acquiring a division, the buyer uses as a benchmark the value that the division can potentially create.[24] As the two divisions have the same production parameters, we set $\ell_a/q^a_{FB} = \ell_b/q^b_{FB} = 0.6$.[25]

As in BCW, we set the annual risk-free rate to $r = 6\%$, the proportional cash-carrying cost to $\lambda = 1\%$, the proportional financing cost to $\gamma = 6\%$, and the fixed financing cost to $\phi = 1\%$ of the firm's total capital stock. With these parameter values, the first-best average q in the neoclassical model is $q_{FB} = 1.4$ and

[24] In this regard we differ from BCW, who assume that the liquidation value is proportional to the book value of the division's capital stock.

[25] In Sections V.C and D, we consider alternative values of the liquidation parameters.

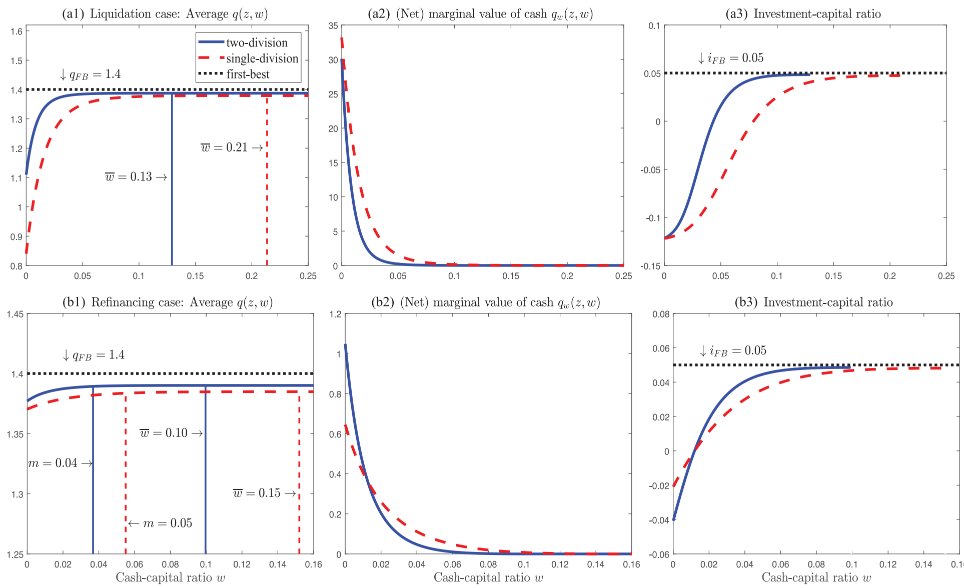

Figure 1. Comparison of a diversified firm (with $z = 0.5$) and a single-division firm. The top panels (A1 to A3) pertain to the liquidation case, the bottom panels (B1 to B3) to the refinancing case. In Panel A1, the vertical lines mark the payout boundary $\overline{w}$. In Panel B1, the vertical lines mark the payout boundary $\overline{w}$ and the equity issuance amount m. (Color figure can be viewed at wileyonlinelibrary.com)

the corresponding first-best investment-to-capital ratio is $i_{FB} = 0.05$ in both divisions.[26]

A. *Diversified versus Stand-Alone Firm*

In Figure 1, we compare the diversified firm (i.e., the firm with the two symmetric divisions described above) with a stand-alone firm. Both firms have the same total capital stock and the same parameter values. The only difference is that the diversified firm has an internal capital market (allowing headquarters to reallocate resources across the two symmetric divisions with imperfectly correlated productivity shocks) and the option to spin off a division at any time it chooses. We assume that the diversified firm has two divisions of equal size ($z = 0.5$).

In Figure 1, the upper panels (A1 to A3) pertain to the liquidation case, where we assume that if the firm runs out of cash, the only option is to liquidate a division or the entire firm (i.e., the firm cannot issue external financing). Alternatively, this can be seen as an extreme case of costly external financing—

[26] The first-best investment-to-capital ratio is given by $i_{FB} = r + \delta_s - \sqrt{(r+\delta_s)^2 - 2(\mu_s - (r+\delta_s))/\theta_s}$, and the first-best q by $q_{FB} = 1 + \theta_s i_{FB}$ under standard convergence conditions for Hayashi-type models.

the cost is so high that firms do not resort to it. We relax this assumption in the lower panels (B1 to B3), which pertain to the refinancing case, where firms have the option to issue equity to replenish their liquidity. In what follows, we discuss both cases, starting with the liquidation case.

Liquidation case. In Panel A1, we plot the average q for the diversified firm (solid line) and stand-alone firm (dashed line), along with the first-best benchmark implied by the neoclassical model, which is $q_{FB} = 1.4$ for our parameter values (dotted line). The horizontal axis plots the cash-to-capital ratio w. The vertical lines mark the endogenous payout boundary at which the firm pays out cash to shareholders ($\overline{w} = 0.13$ for the diversified firm and $\overline{w} = 0.21$ for the stand-alone firm).

When the firm runs out of cash (reaching $w = 0$), the diversified firm spins off one of the divisions.[27] Since the divisions are symmetric and the firm starts with $z_0 = 0.5$, the investment levels of both divisions are identical at all times, and hence $z_t = 0.5$ for all t before the firm liquidates a division. (For this reason, the choice of which division to liquidate is immaterial.) Assuming that division a is spun off, the firm receives the liquidation value $\ell_a K^a$ and becomes a single-division firm with a cash-to-capital ratio of $\frac{\ell_a K^a}{K^b} = \ell_b = 0.84$. Since this ratio exceeds the dividend payout boundary of the single-division firm ($\overline{w} = 0.21$), it will optimally pay out the difference of 0.63 to shareholders, and then operate as a stand-alone firm with the remaining liquidity.

Note that this sequence is consistent with Dittmar's (2004) finding that firms often pay a special dividend subsequent to a spinoff. Given this optimal response, the diversified firm's value at $w = 0$ is 1.11.[28] In contrast, when $w = 0$, the stand-alone firm has no choice but to liquidate the entire firm. Given our assumption that the liquidation value is 0.6 times the (first-best) market value, this implies that the stand-alone firm's value at $w = 0$ is $0.6 \times q_{FB} = 0.84$. As these calculations illustrate, liquidation is more costly for the stand-alone firm (as it permanently forgoes all future growth opportunities) compared to the diversified firm (as it liquidates a division in lieu of the entire firm).

As can be seen, we find that the diversified firm achieves a higher valuation compared to the stand-alone firm, especially in bad times when the firm is low on cash.[29] The rationale is twofold. First, the diversified firm has the option to spin off a division to replenish its liquidity. This option is more valuable for lower values of w.[30] Second, diversification reduces the volatility of the

[27] For our parameter values, it is never optimal to liquidate a division when $w > 0$. In Section V.D, we consider alternative parameterizations, under which early liquidation can be optimal. Note that, as shown in Section IV.C, it is never optimal for the diversified firm to liquidate both divisions at once.

[28] Formally, the diversified firm's value at $w = 0$ satisfies $F(K^a, K^b, W) = P^b(K^b, \ell_a K^a)$, and hence $f(z, w) = (1 - z)p^b(\ell_a)$. For $z = 0.5$ and our parameter values, $f(0.5, 0) = 1.11$.

[29] This pattern is consistent with the empirical evidence of Matvos and Seru (2014) and Kuppuswamy and Villalonga (2016), who find that the value of diversification was higher during the recent financial crisis. It also echoes Matvos, Seru, and Silva's (2018) finding that firms aim to diversify their operations in times of capital market disruptions.

[30] Indeed, this spinoff option makes $q(z, w)$ at $w = 0$ exceed the liquidation value for the stand-alone firm, $\ell = 0.6 \times 1.4 = 0.84$.

firm's productivity shocks and hence the likelihood of costly liquidation. This benefit from diversification is especially valuable in bad times when w is low.[31] We further observe that the payout boundary $\overline{w}$ is lower for the diversified firm. The same two rationales explain this finding, that is, the conglomerate's option to spin off a division and the lower volatility of the productivity shocks reduce the value of holding cash. As such, the diversified firm can more easily afford to pay a dividend. This finding is consistent with the evidence of Duchin (2010), who documents that diversified firms hold less cash compared to stand-alone firms.

Panel A2 plots the (net) marginal value of cash q_w. As is shown, the marginal value of cash increases as the firm becomes more constrained and liquidation more likely. Since liquidation is more costly for the stand-alone firm, the marginal value of cash is higher for the stand-alone firm compared to the diversified firm. Note that costly liquidation induces both firms to be de facto risk averse, as the average q of both firms is concave in w. Thus, holding cash today (below the payout boundary $\overline{w}$) maximizes firm value and reduces the likelihood of liquidation in the future. Intuitively, cash is valuable as it helps keep the firm away from costly liquidation.

Panel A3 plots the investment-to-capital ratio for both firms, along with the first-best level ($i_{FB} = 0.05$ for our parameter values). Due to financing constraints, investment is below the first-best for both firms. Importantly, underinvestment is more severe for the stand-alone firm. This mirrors the pattern in Panel A2. As liquidity is more valuable for the stand-alone firm (compared to the diversified firm), it has greater demand for cash and hence reduces investment more aggressively.[32]

Refinancing case. In the lower panels of Figure 1, we consider the refinancing case. Specifically, when the firms run out of cash ($w = 0$), they now replenish their liquidity by raising external equity. Doing so is costly, as the firms incur fixed ($\phi = 1\%$) and variable ($\gamma = 6\%$) financing costs. In Panel B1, we plot the average q for both the diversified and single-division firms. Because the firms can now issue equity (at a cost), they avoid inefficient liquidation even under financial distress. As a result, for both firms, the average q is higher in the refinancing case compared to the liquidation case in Panel A1. Moreover, the respective payout boundaries ($\overline{w}$) are lower than in the liquidation case. This is because firms are more willing to pay out cash when they can raise new funds in the future.

We continue to find that the diversified firm is more valuable than the stand-alone firm. As diversification reduces the volatility of the firm's cash flows, it reduces the likelihood of running out of cash and resorting to (costly) equity issuance. As a result, the diversified firm has higher value, can more easily

[31] Note that our setup is conservative in that it likely underestimates the value gains from diversification. This is because of our assumption of i.i.d. productivity shocks. In reality, shocks are likely to exhibit some degree of persistence, which increases the benefits from diversification.

[32] Note that close to $w = 0$, investment is negative for both firms (and even more so for the stand-alone firm). Intuitively, the firms disinvest to raise cash and stay away from the liquidation boundary.

afford to pay out cash ($\overline{w} = 0.10$ for the diversified firm, and $\overline{w} = 0.15$ for the stand-alone firm), and issues less equity in the event of refinancing ($m = 0.04$ for the diversified firm, and $m = 0.05$ for the stand-alone firm).

In Panels B2 and B3, we find that when liquidity is abundant, the diversified firm is less prone to underinvestment and has a lower marginal value of cash, compared to the stand-alone firm. This is consistent with our previous analysis for the liquidation case. As diversification reduces the volatility of the firm's cash flows, the diversified firm has less of a need for liquidity (i.e., liquidity is less valuable) and is more inclined to invest instead of hoarding cash.

Interestingly, the opposite results hold when w is low (the two curves cross in both Panels B2 and B3). The intuition is as follows. As the volatility of the firm's cash flows is reduced through diversification, an extra dollar of cash becomes more effective in helping the diversified firm avoid costly equity issuance. When w is sufficiently low, the (marginal) value of cash is larger for the diversified firm than the stand-alone firm, as the value-add of preserving both divisions is very high. Therefore, the diversified firm reduces investment more than the single-division firm. As a result, diversification can lead to a paradoxically higher demand for precautionary savings, and hence more underinvestment, when the cash situation is sufficiently dire.

B. Relative Size of Divisions: z

In Figure 1, we consider a diversified firm with equal-sized divisions ($z = 0.5$). Note that in this (symmetric) case, $z = 0.5$ is an absorbing state.[33] Therefore, the solution for the diversified firm boils down to a single state variable (w) problem as in BCW but with lower volatility (due to the imperfect correlation between the two divisions' productivity shocks).

Albeit insightful, the symmetric-division case with $z = 0.5$ is a rather special case. In what follows, we examine the case in which $z \neq 0.5$. As we will see, a key insight from this analysis is that diversification can reduce firm value in low-liquidity states, as it increases the cost of a spinoff (which peaks at $z = 0.5$, ceteris paribus) and hampers liquidity management.

Liquidation case. In Figure 2, we analyze the liquidation case with $z = 0.1$, which means that division a accounts for 10% of the firm's capital stock, while division b accounts for the remaining 90%.

Panel A plots the value of the firm for the $z = 0.1$ case, and compares it with the $z = 0.5$ case analyzed earlier. When cash is abundant (high w), the balanced firm ($z = 0.5$) is more valuable. This finding is intuitive—the diversification gains are highest at $z = 0.5$, which translates to higher firm value. In contrast, in bad times (low w), the balanced firm is less valuable. This is because, closer to $w = 0$, liquidation is more likely, and liquidation is more costly when $z = 0.5$, as the firm would forgo half of its productive assets. In contrast, in the event

[33] For the case in which the two divisions are symmetric, when $z_t = 0.5$, we have $i_t^a = i_t^b$ and $\delta_a = \delta_b$, which imply $dz_t = 0$, as can be seen from (43).

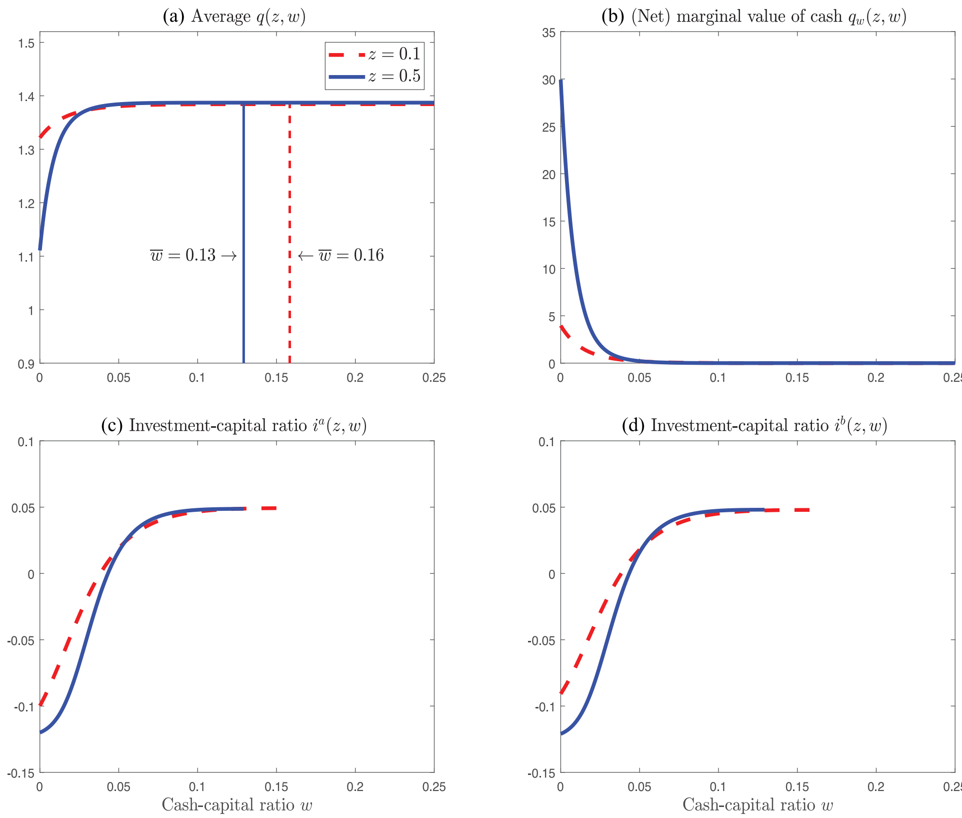

Figure 2. Liquidation case—comparison of diversified firms with $z = 0.1$ and $z = 0.5$. In Panel A, the vertical lines mark the payout boundary $\overline{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

of liquidation, the $z = 0.1$ firm would optimally spin off the smaller division (division a) and only forgo 10% of its productive assets.

Panel B plots the marginal value of cash, and Panels C and D the investment-to-capital ratio for divisions a and b, respectively. The observed patterns are consistent with the above interpretation. In high-w states, liquidity is less valuable for the more balanced firm, as it faces lower volatility due to diversification. Given the lower need for cash, it is able to invest more compared to the less balanced firm ($z = 0.1$). In contrast, in low-w states, firms worry about liquidation. Since liquidation is more costly to the more diversified firm ($z = 0.5$), it is more eager to prevent this scenario from happening. As a result, compared to the $z = 0.1$ firm, the more diversified firm reduces investment more aggressively to preserve cash, and cash has a higher marginal value.

Overall, the patterns from Figure 2 imply both a dark side and a bright side of diversification from the perspective of liquidity management, even for

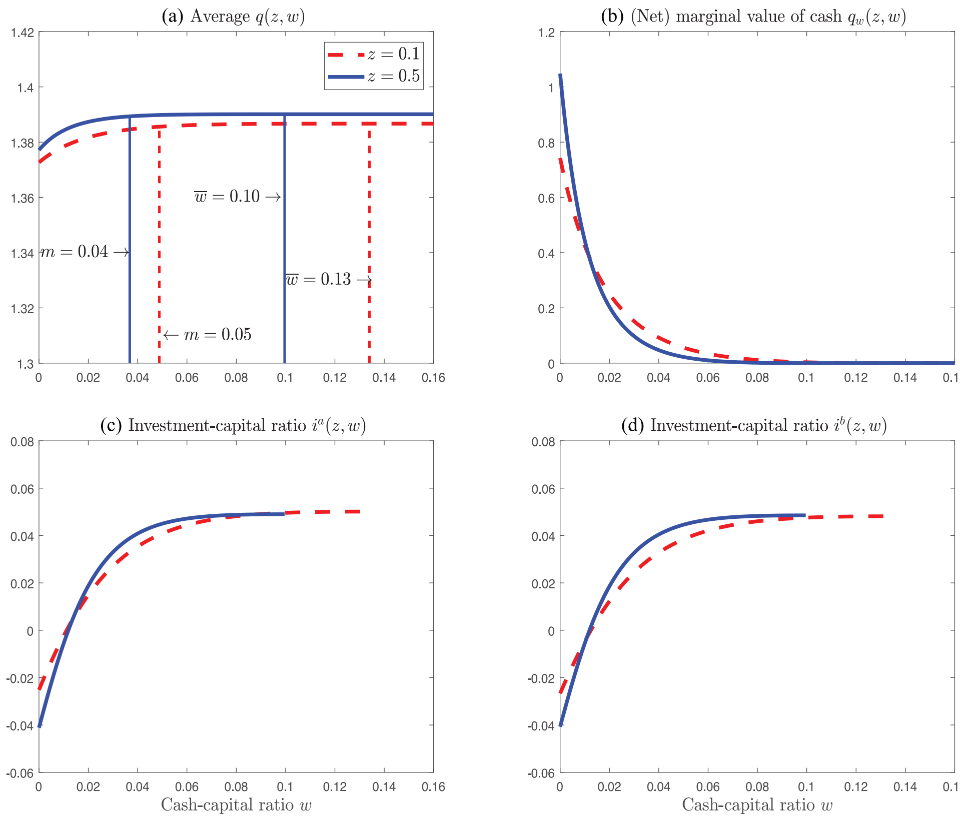

Figure 3. Refinancing case—comparison of diversified firms with $z = 0.1$ and $z = 0.5$. In Panel A, the vertical lines mark the optimal payout boundary $\overline{w}$ and the equity issuance amount m. (Color figure can be viewed at wileyonlinelibrary.com)

a value-maximizing conglomerate. In good times, diversification reduces the need for liquidity and creates value. In low-liquidity states, diversification can hamper the conglomerate's liquidity management by making spinoffs more costly and destroys value.

Refinancing case. In Figure 3, we analyze the refinancing case. When equity financing is less costly than liquidation, both firms choose to issue equity when they run out of cash ($w = 0$). Thus, the more balanced firm ($z = 0.5$) no longer bears the higher liquidation costs arising from the liquidation of a relatively large division. As a result, the benefits from diversification—that is, the lower volatility of the balanced firm's cash flows—dominate, and the more balanced firm is always more valuable than the $z = 0.1$ firm, as shown in Panel A.

Interestingly, Panel B shows that the marginal value of cash is lower for the more balanced firm in good times (high w), but higher in bad times (low w). Moreover, as shown in Panels C and D, the more balanced firm cuts investment in low-w states.

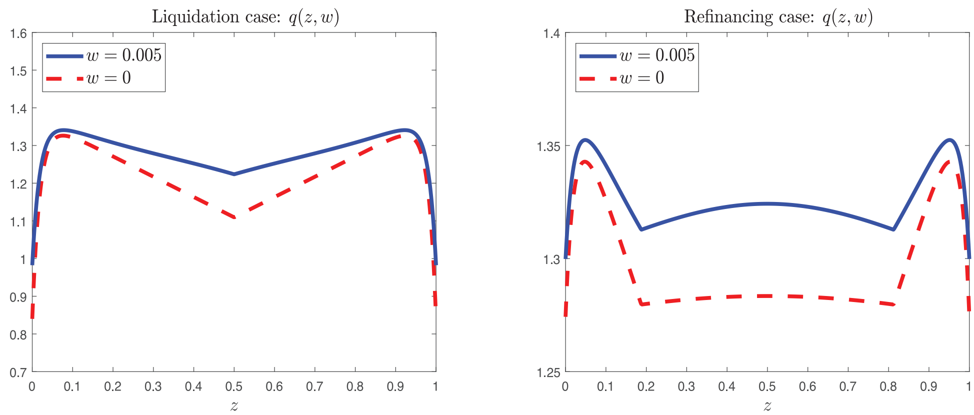

Figure 4. Average q in the liquidation (Panel A) and refinancing (Panel B) cases. In Panel B, the fixed financing cost is set to $\phi = 0.1$. (Color figure can be viewed at wileyonlinelibrary.com)

While this pattern mirrors that in Figure 2, the rationale is different for low w. In bad times—due to the higher degree of diversification—an extra dollar of cash is more effective in helping the more balanced firm avoid issuing costly equity. As a result, the more balanced firm has a stronger preference for liquidity closer to $w = 0$.

Diversification can reduce value. As we show above when comparing a well-diversified firm ($z = 0.5$) and a less diversified firm ($z = 0.1$), diversification can reduce value in low-liquidity states. In Figure 4, we examine this relationship more systematically by plotting $q(z, w)$ for $w = 0.005$ (i.e., when the firm is almost out of liquidity, blue solid line) and $w = 0$ (when the firm is out of liquidity, red dashed line). Panels A and B correspond to the liquidation and refinancing case, respectively. As is shown, for both $w = 0.005$ and $w = 0$, the well-diversified firm ($z = 0.5$) can be less valuable than less diversified firms for a broad range of z values.[34]

This reinforces our point that diversification can reduce firm value in low-liquidity states, as it increases the cost of a spinoff and hampers liquidity management.

C. Characterization of Solution Regions

In Figure 5, we characterize our model's solution by regions for the liquidation case (Panel A) and two variants of the refinancing case (Panels B and C, along with the respective equity issuance amount m reported in Panel D). As we have two state variables, namely, the cash-to-capital ratio w and the relative size z of division a, all regions are defined by (w, z). The horizontal and

[34] Also note that the conglomerate's average q is nonmonotonic in z. Consider, for example, the liquidation case (Panel A). For the $w = 0.005$ case, $q(z, w)$ peaks at $z = 0.08$. As z decreases from $z = 0.08$ toward zero, the firm is effectively becoming a single-division firm, which is less valuable. This is because the conglomerate is very likely to lose its diversification benefit.

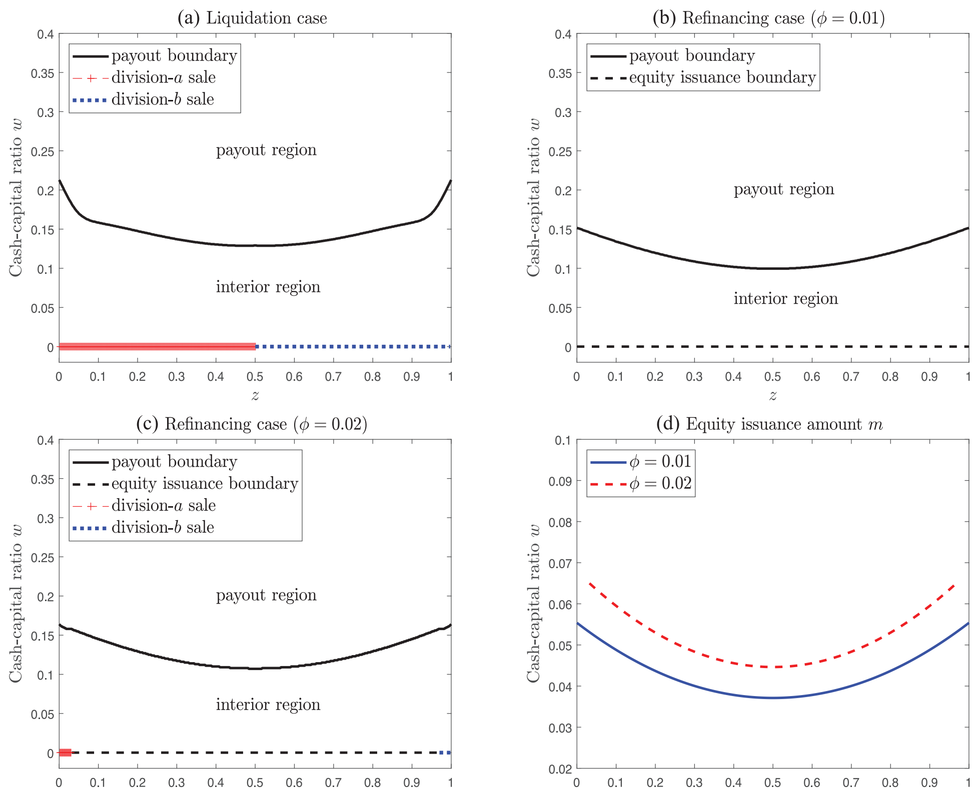

Figure 5. Solution regions for a firm with two symmetric divisions in the liquidation (Panel A) and refinancing (Panels B and C) cases. Panel D plots the equity issuance amount m. (Color figure can be viewed at wileyonlinelibrary.com)

vertical axes correspond to z and w, respectively. (When $z = 0$ or $z = 1$, the firm is a stand-alone firm.) The solid line corresponds to the payout boundary $\overline{w}$ as a function of z, $\overline{w}(z)$. The function $\overline{w}(z)$ is part of the "payout region" and separates this region from the "interior region." If $w_t \geq \overline{w}(z_t)$, the firm is in the payout region and pays out its excess cash $w_t - \overline{w}(z_t)$ to the shareholders.

Liquidation case. In Panel A, we consider the liquidation case. As can be seen, the payout boundary is lowest when the firm's z reaches the absorbing state $z = 0.5$, and increases as the firm becomes less balanced. Intuitively, since the volatility of the firm's cash flows is lowest at $z = 0.5$, the balanced firm has the lowest demand for precautionary savings; the more unbalanced the firm is, the higher the volatility, and the higher the demand for precautionary savings.

When the firm runs out of cash ($w = 0$), it hits the liquidation boundary. Since liquidation is more costly for larger divisions (as the firm forgoes more of its productive assets), the firm optimally liquidates the smaller of the two divisions, that is, the firm liquidates division a when $z < 0.5$ (represented by the line with the + markers) and division b when $z > 0.5$ (dotted line).[35] This prediction is consistent with the empirical evidence of Maksimovic and Phillips (2001), who find that multidivision firms are more likely to spin off their smaller divisions.

Refinancing case. In Panel B, we turn to the refinancing case. For our baseline parameters, refinancing is less costly than liquidation. Accordingly, when the firm runs out of cash ($w = 0$), it now hits the refinancing boundary (represented by the dashed line) and responds by issuing equity to replenish its cash. The firm has no incentive to issue equity sooner, as doing so would forgo the option of avoiding costly equity issuance.

We plot the corresponding equity issuance amount m in Panel D (solid line). As is shown, both the payout boundary $\overline{w}$ and the equity issuance amount m are lowest at $z = 0.5$, and are higher the higher the imbalance between the two divisions. The intuition is the same as in the liquidation case—since the balanced firm is better diversified (and hence faces lower volatility), it can more easily afford to pay a dividend and needs less financing conditional on issuing equity.

In Panel C, we depart from our baseline parameters by assuming that equity financing has a higher fixed cost ($\phi = 2\%$). In this case, we find that refinancing is not always preferred to liquidation. When one of the divisions is sufficiently small (specifically, when $z < 0.03$ or $z > 0.97$), liquidation is less costly (as the firm only forgoes a relatively small fraction of its productive assets), and the firm prefers to spin off the smaller division as opposed to issuing costly equity. When $z \in [0.03, 0.97]$, the firm issues equity when it exhausts its cash holdings.

Finally, we observe in Panel D that the equity issuance amount is always higher when $\phi = 2\%$ (compared to the baseline case with $\phi = 1\%$). Due to the higher fixed costs of issuing equity, firms resort to higher amounts to reduce the odds of bearing the fixed costs again in the future.

Note that our finding that the firm may find both external financing and division sale (liquidation) to be optimal on the equilibrium path is not possible in BCW. This is because our model features two divisions, and whether equity financing or liquidation is optimal for a financially distressed firm depends on z (in addition to w). This result illustrates how, at the conceptual level, the analysis of a financially constrained multidivision firm can be fundamentally different compared to that of a single-division firm.

D. Early Liquidation

With two symmetric divisions (i.e., the structural parameters μ, σ, δ, θ, and ℓ are the same for both divisions), liquidating a division when $w > 0$ ("early" liquidation) is never optimal. However, with asymmetric divisions, early liquidation can be optimal.

[35] When $z = 0.5$, the firm is indifferent between spinning off either division a or b.

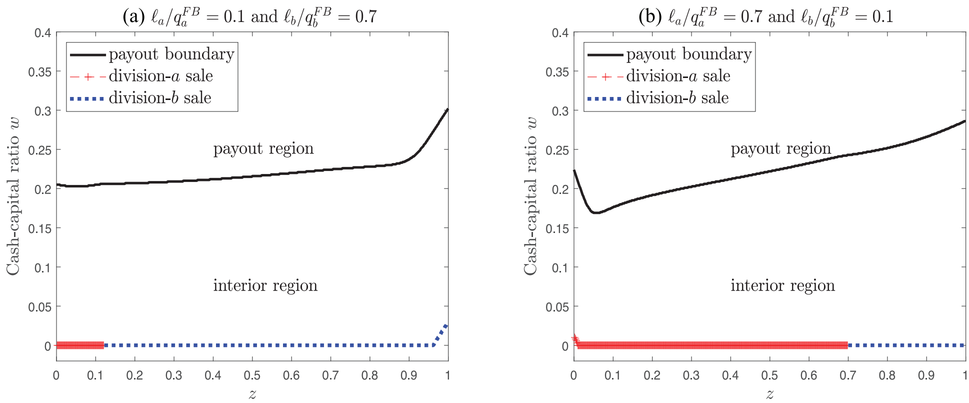

Figure 6. Early liquidation. This figure plots the solution regions for a diversified firm with asymmetric divisions with $\mu_a = 24\%$, $\mu_b = 16\%$, and $\rho = 0.9$. In Panel A, the more productive division (division a) has a lower liquidation value ($\ell_a/q^a_{FB} = 0.1$ and $\ell_b/q^b_{FB} = 0.7$). In Panel B, it has a higher liquidation value ($\ell_a/q^a_{FB} = 0.7$ and $\ell_b/q^b_{FB} = 0.1$). (Color figure can be viewed at wileyonlinelibrary.com)

We consider such a parameterization in Figure 6. Specifically, we assume that the two divisions have different productivity ($\mu_a = 24\%$ and $\mu_b = 16\%$) and different liquidation values. Moreover, we set the correlation coefficient between the divisional productivity shocks to $\rho = 0.9$ so that the diversification benefits are lower than in our baseline analysis. The other parameters are the same as in Table I pertaining to the firm with symmetric divisions.

In Panel A, we consider the case in which the more productive division (division a) has a lower liquidation value than the other division ($\ell_a/q^a_{FB} = 0.1$ and $\ell_b/q^b_{FB} = 0.7$). In this case, it is optimal for the firm to voluntarily liquidate division b before running out of cash. This is because the diversification benefit of keeping division b is limited (due to the high ρ and small size $1 - z$ of division b), and the cost of liquidating division b is relatively low (due to the relatively high liquidation value ℓ_b/q^b_{FB} and small size $1 - z$ of division b). Both forces make the firm willing to liquidate early to enhance its liquidity and mitigate underinvestment going forward, especially in low-w states.

In Panel B, we turn to the more realistic scenario in which the more productive division (division a) also has a higher liquidation value ($\ell_a/q^a_{FB} = 0.7$ and $\ell_b/q^b_{FB} = 0.1$). The firm now liquidates division a before running out of cash when division a is sufficiently small (z close to 0). Note that, in this case, the firm liquidates the division with the *higher* productivity in low-w states.[36]

[36] This prediction is consistent with the empirical evidence of Schlingemann, Stulz, and Walkling (2002), who find that firms in need of cash tend to liquidate their more liquid divisions (i.e., their divisions with higher liquidation values), even if those are their more productive units. Relatedly, this prediction is consistent with the empirical evidence of Ma, Tong, and Wang (2021), who find that in financial distress, innovative firms are more likely to liquidate their most productive assets (specifically, their core patents) if doing so allows them to raise more cash.

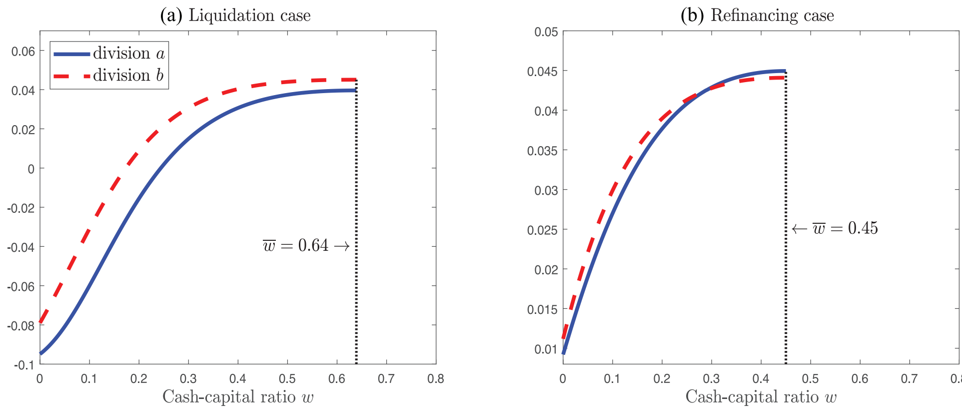

Figure 7. Resource allocation with different volatilities. This figure plots division-specific investment-to-capital ratios as a function of w (fixing $z = 0.5$) for a firm with asymmetric divisions with $\mu_a = 20.5\%$ and $\sigma_a = 40\%$ for division a, and $\mu_b = 19.5\%$ and $\sigma_b = 9\%$ for division b, respectively. The dotted line marks the payout boundary $\overline{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

This is because liquidating the division with a higher liquidation value generates more cash, which is highly valuable in low-w states, even though doing so eliminates the division with higher productivity.

Importantly, this finding illustrates that when taking risk management considerations into account, it can be misleading to rank divisions solely based on their productivity (μ). We discuss this point in more detail in the next section.

E. Resource Allocation with Different Volatilities

In Figure 7, we examine the resource (re-)allocation across two asymmetric divisions. A real-world relevant case is when one division has high μ and high σ, while the other has low μ and low σ, which provides an economically meaningful trade-off. We set $\mu_a = 20.5\%$ and $\sigma_a = 40\%$ for division a, and $\mu_b = 19.5\%$ and $\sigma_b = 9\%$ for division b. All other parameters are the same as in our baseline, and we consider policies at $z = 0.5$.

Panel A plots the investment-to-capital ratio for both divisions in the liquidation case. Perhaps surprisingly, headquarters channels more of the firm's resources toward the low-μ and low-σ division, as opposed to the high-μ and high-σ division. This pattern is especially pronounced when the firm is low on cash (low w). Intuitively, the firm prefers to reduce its risk exposure and preserves its going-concern value by investing in the low-μ and low-σ division, especially in low-w states for which the likelihood of liquidation is higher.

Panel B considers the refinancing case. As can be seen, the optimal capital allocation differs depending on w. In bad times (low w), more resources are allocated toward the low-σ division. The intuition is analogous to the liquidation case. In low-w states, the firm's primary concern is to manage risk and reduce the probability of tapping costly external equity financing, rather than generate higher expected cash flows. In contrast, in good times (high w), headquarters allocates more of the firm's resources toward the high-μ division—as in static models of winner picking (e.g., Stein (1997)). The reason we obtain the static model intuition in high-w states is because, when cash is abundant, profit-generating considerations dominate risk management considerations. Conversely, the reason the two lines do not cross in Panel A is because liquidation is too costly and hence risk management considerations dominate for all levels of w.

These findings highlight the importance of considering a dynamic setting in characterizing the mechanics of internal capital markets. In static models of winner picking, headquarters allocates resources to the more productive (high-μ) units. In a dynamic setting, firms can run out of cash and therefore need to engage in risk management. As our model shows, when cash is scarce, headquarters may optimally channel resources toward the low-risk division (i.e., based on σ). These considerations motivate a broader formulation of the winner-picking role of internal capital markets: When headquarters allocates resources to divisions, it does so based not only on productivity, but also on risk.[37]

From this perspective, the within-firm allocation of resources can be viewed as a dynamic portfolio choice problem where different divisions offer very different risk-return trade-offs. Unlike the standard portfolio choice problem (e.g., Merton (1971)), the risk-neutral firm in our model is endogenously risk averse, and the instruments used by the firm (e.g., equity issuance) are different from those used by households. As described above, this endogenous risk aversion depends on both the firm's liquidity (w) and the size distribution of its divisions (z).

Our findings have important implications for capital budgeting. Standard corporate finance textbooks prescribe that capital budgeting should be done based on the weighted average cost of capital (WACC) of the stand-alone project, and the project's idiosyncratic risk should not matter. Our framework shows that this prescription is misguided. Indeed, ignoring idiosyncratic risk and the balance sheet of the firm when doing capital budgeting is incorrect; depending on the firm's liquidity, costs of external financing, and liquidation costs, it may be optimal to invest in a lower NPV project (based on the project's WACC) if the project's idiosyncratic risk is sufficiently low. Adding a new project (or a new division) changes the firm's entire balance sheet composition and risk profile. As such, the firm should evaluate the new project by computing the net value difference caused by introducing the new project into the firm, as opposed to evaluating the project as if it were a stand-alone project.[38]

[37] Note that the risk-return trade-off at the divisional level cannot be fully characterized by the division's μ/σ ratio due to the nonlinear nature of the model. Section IV.A of the Internet Appendix illustrates this point by reproducing Figure 7, but setting μ_a, μ_b, σ_a, and σ_b to half of their respective values (such that μ_a/σ_a and μ_b/σ_b are unchanged), and shows that the division-specific investment-to-capital ratios are not identical to those in Figure 7.

[38] Consider a setting in which the unconditional Capital Asset Pricing Model (CAPM) holds under MM (i.e., for a financially unconstrained firm). For a financially constrained firm, the uncon-

F. Retention of Loss-Making Divisions

Our previous analysis shows that risk management considerations can induce conglomerates to optimally channel more resources toward the low-productivity division. In fact, conglomerates may even choose to retain a loss-making division (i.e., a division with negative productivity, $\mu < 0$) if the diversification benefits are so large that they outweigh the firm's productivity losses. We consider such a case in Section IV.B of the Internet Appendix. Specifically, we show that a large negative correlation ρ between the two divisions can induce the conglomerate to optimally retain a loss-making division.

G. Debt Financing

In Section V of the Internet Appendix, we show that our predictions continue to hold if we allow for debt financing, which we model via a credit line as in BCW. This analysis further shows that extending a line of credit to the conglomerate creates value, lowers the marginal value of cash, mitigates underinvestment, and accelerates payouts by lowering the payout boundary. Moreover, we show that the quantitative effects are substantial.

VI. Quantitative Analysis: Socialism

In this section, we provide a quantitative analysis for our model with socialism.[39] Corporate socialism is economically interesting when the divisions have different productivity. Specifically, we assume that division a is more productive than division b. As described in Section VI.III, corporate socialism makes division a less productive and division b more productive, with the net effect being negative, in that the conglomerate as a whole is less productive.

We first consider the case in which socialism surfaces only for the ongoing operations (Section A). We then analyze the case in which socialism also matters for the spinoff decision (Section VI.B). To ease exposition, we analyze the liquidation case.[40]

Parameter choices. We set the annual productivity of the two divisions to $\mu_a = 0.24$ and $\mu_b = 0.16$. As corporate socialism generates a (scaled) cost of $g_c(z) = \theta_c(\mu_a - \mu_b)z(1-z)$ per unit of time, we set the socialism parameter to $\theta_c = 1$, so that the maximum socialism cost is $(0.24 - 0.16) \times 0.5 \times (1 - 0.5) = 2\%$ of firm size per annum. The other parameters are the same as in our baseline analysis. The full set of parameters used in this section is provided in the last column of Table I.

ditional CAPM does not hold but instead the conditional CAPM holds as the firm is endogenously risk averse (BCW). With financing constraints and multiple divisions, both w and z affect the firm's risk-return trade-off, and hence influence corporate investment and capital budgeting decisions. As a result, the firm's cost of capital depends on both w and z for a multidivision firm. We can show that a conditional CAPM—where beta depends on both w and z—holds for the financially constrained multidivision firm.

[39] Section VI.A of the Internet Appendix provides the model solution.

[40] In Section VI.B of the Internet Appendix, we analyze the refinancing case.

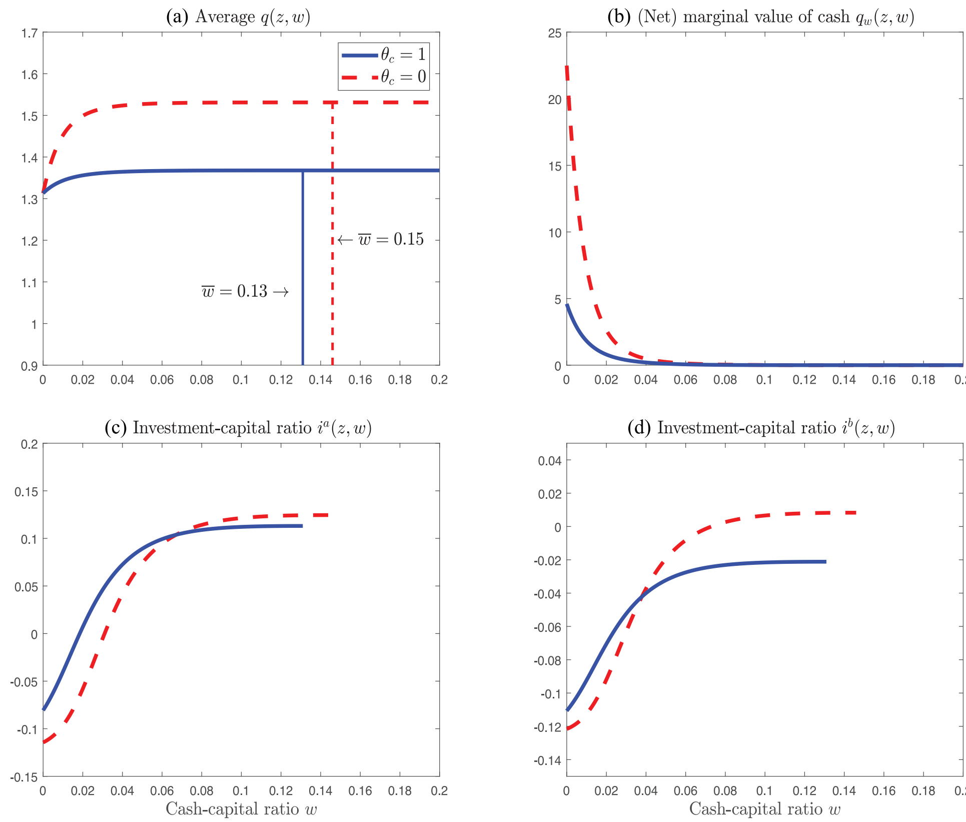

Figure 8. Comparison of diversified firms with and without socialism in the liquidation case. The firms have asymmetric divisions with $\mu_a = 24\%$ and $\mu_b = 16\%$. The divisions are of equal size ($z = 0.5$). In Panel A, the vertical lines mark the payout boundary $\overline{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

A. Socialism: Ongoing Operations Only

In Figure 8, we consider the liquidation case with (solid line) and without (dashed line) socialism, for a firm with balanced divisions ($z = 0.5$). In Panel A, we find that the value of the firm is always lower with socialism, consistent with the dark side models of internal capital markets. This finding is intuitive. Influence activities reduce the firm's productivity, which in turn reduces the value of the firm. Moreover, we find that the payout boundary is lower with socialism. As socialism reduces the firm's productivity, it makes payout more desirable from the shareholders' perspective.

Interestingly, we find that the difference in valuation shrinks as we move closer to the liquidation boundary ($w = 0$). This reflects the lower cost of liquidating a division when the firm is subject to socialism—by spinning off a division, the firm becomes a stand-alone firm that is free of socialism frictions. Accordingly, the option to spin off a division is more valuable with socialism

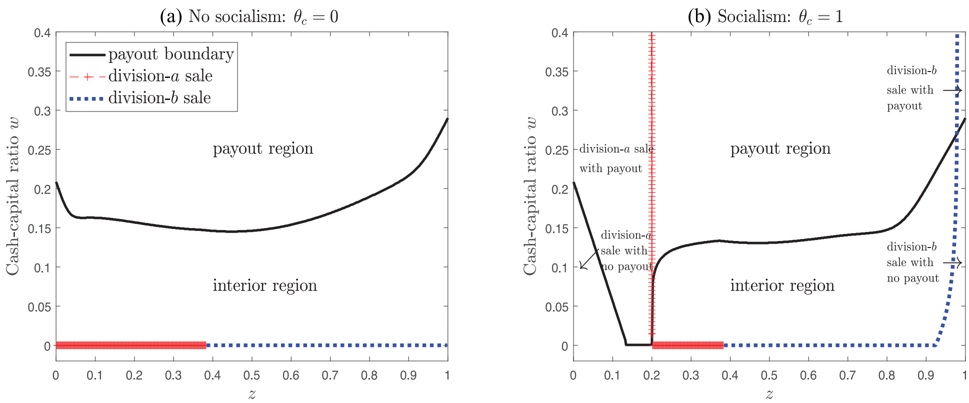

Figure 9. Comparison of policy regions for diversified firms with and without socialism in the liquidation case. The firms have asymmetric divisions with $\mu_a = 24\%$ and $\mu_b = 16\%$. (Color figure can be viewed at wileyonlinelibrary.com)

than without. Indeed, when $w = 0$, spinning off one division fully addresses the socialism problem, and hence the average q with socialism at $w = 0$ is the same as without socialism ($q = 1.31$).

This insight has implications for liquidity management. With socialism, it is less costly to replenish the firm's liquidity through a spinoff. As a result, liquidity is less valuable with socialism than without, especially when the firm is low on cash. This is consistent with the pattern in Panel B, showing that the marginal value of cash is lower with socialism, especially in low-w states. Note that the quantitative effect is quite large—for example, near $w = 0$, the (net) marginal value of cash $q_w(z, w)$ drops from 22.5 to 4.6 due to socialism. Relatedly, the patterns in Panels C and D show that, with socialism, the firm is less prone to underinvestment in low-w states (as it has less of a need to preserve cash by reducing investment). In contrast, in high-w states, the lower productivity (due to socialism frictions) dominates, such that investment is lower for the firm with socialism.

In Figure 9, we plot the solution regions. Without socialism (Panel A), the solution regions are similar to those in Panel A of Figure 5, except that they are no longer symmetric around $z = 0.5$ due to the higher productivity of division a (compared to division b). In particular, when the firm runs out of cash ($w = 0$), it is now less likely to liquidate division a relative to division b. That is, the firm liquidates division a only when it exhausts its cash ($w = 0$) and when it is sufficiently small compared to division b ($z \leq 0.38$). In our numerical example, the payout boundary for a stand-alone firm with only division b (when $z = 0$) is $\overline{w}(0) = \overline{w}_b = 0.21$, where $\overline{w}_b$ is the optimal payout boundary for a stand-alone firm with only division b, as in BCW.[41]

[41] Similarly, with only division a (when $z = 1$), $\overline{w}(1) = \overline{w}_a = 0.29$, where $\overline{w}_a$ is the optimal payout boundary for a stand-alone firm with only division a.

Panel B of Figure 9 reports the solution regions with socialism. There are six regions in total. To the left of the red dashed line are the two regions in which the firm sells its more productive division a. Whether the firm pays a dividend upon selling division a depends on the level of w for a given level of z. The dividing line between these two regions (involving the sale of division a) is given by the downward-sloping linear function

$$\overline{w}(z) = \max\{(1-z)\overline{w}_b - \ell_a z, 0\}, \tag{58}$$

where $\overline{w}_b$ is the optimal payout boundary for a stand-alone firm with only division b. As socialism disappears when $z = 0$, we have $\overline{w}(0) = \overline{w}_b = 0.21$, which is the same as in Panel A without socialism (when $\theta_c = 0$).

If (z, w) is in the division-a sale region and w is sufficiently large, in that $w > \overline{w}(z)$, the firm makes a one-time dividend payment $w - \overline{w}(z)$ per unit of the conglomerate's capital stock, liquidates division a, and then operates as a stand-alone firm with the remaining division b.[42] Otherwise (i.e., if $w < \overline{w}(z)$), the firm pays no dividend when liquidating division a. These two regions are marked in Panel B as "division-a sale with payout" and "division-a sale with no payout," respectively.

To the right of the blue dotted line are the two regions in which the firm sells its less productive division b. Whether the firm pays dividends upon selling its division b depends again on the level of w for a given level of z. The dividing line between these two regions (involving the sale of division b) is given by the upward-sloping linear function

$$\overline{w}(z) = \max\{z\overline{w}_a - \ell_b(1-z), 0\}, \tag{59}$$

where $\overline{w}_a$ is the optimal payout boundary for a stand-alone firm with only division a.[43]

In the middle of Panel B are the two remaining regions, the payout region and the interior region, which are divided by the black solid nonlinear curve. When in the payout region, the conglomerate makes a dividend payment to bring its w vertically down to that curve. In the interior region, the conglomerate chooses its divisional investment levels as a going concern; if the

[42] For example, a diversified firm with $z = 0.05$ and $w = 0.2$ lies in the region marked by "division-a sale with payout." The payout threshold at $z = 0.05$ corresponds to $\overline{w}(0.05) = (1 - 0.05) \times \overline{w}_b - \ell_a \times 0.05 = 0.13$. (Recall that $\overline{w}_b = 0.21$ and $\ell_a = (\ell_a/q^a_{FB}) \times q^a_{FB} = 0.6 \times 2.2$.) The firm makes a one-time dividend payment $w - \overline{w}(0.05) = 0.2 - 0.13 = 0.07$ per unit of the conglomerate's capital stock, liquidates division a, and then operates as a stand-alone firm with a scaled cash balance of $\overline{w}_b = 0.21$.

[43] If (z, w) is in the division-b sale region and w is sufficiently large, in that $w > \overline{w}(z)$, the firm makes a one-time dividend payment $w - \overline{w}(z)$ when liquidating division b. Otherwise (i.e., if $w < \overline{w}(z)$), the firm pays no dividend when liquidating division b. These two regions are marked in Panel B as "division-b sale with payout" and "division-b sale with no payout," respectively.

conglomerate runs out of cash ($w = 0$), it liquidates division a when $0.20 < z < 0.38$ and division b when $0.38 \leq z < 0.92$. Moreover, we see that the firm is less willing to pay out as z increases, as the firm moves closer to being a single-division firm with only the more productive division a. Being more productive and less diversified, the conglomerate has a stronger motive to retain cash inside the firm.

Finally, a comparison of the two panels in Figure 9 provides additional insights. First, the two end points (at $z = 0$ and $z = 1$) have the same values on the vertical axes with and without socialism, as they map into the same single-division firms that are free of socialism. Second, for $z \in (0, 1)$, socialism effectively acts as a tax on the conglomerate's capital stock (in a nonlinear way), reducing its size-weighted productivity. Recall that a less productive firm has lower demand for cash. Therefore, a conglomerate with socialism has less of a need to hold cash. This explains why the payout boundary in Panel B (with socialism) is lower than in Panel A (without socialism). Third, socialism makes division sale more attractive as the conglomerate becomes less productive. As a result, Panel B features a "division-a sale with payout" region (top left) and a "division-b sale with payout" region (top right).

B. Socialism: Both Ongoing Operations and Division Sale

In Figure 10, we compare the solution for the case in which socialism influences both ongoing operations and spinoffs ($\theta_d = 3$) with the case (analyzed in Section VI.A) in which socialism influences only ongoing operations ($\theta_d = 0$). As can be seen, when division managers can also influence the spinoff decision, the firm delays its payout (as reflected in the higher $\overline{w}$ in Panel A) and lowers the investment-to-capital ratio for both divisions so as to preserve financial slack (Panels C and D). As a result, the firm's average q (Panel A) is lower and the firm's (net) marginal value of cash $q_w(z, w)$ (Panel B) is higher, especially when the firm is financially constrained (near $w = 0$). Moreover, Panel B shows that the marginal value of cash for the conglomerate near $w = 0$ approaches 20.5—this reflects the firm's strong aversion to spinning off a division as the cost of doing so is now very high due to the division manager's lobbying efforts to prevent the spinoff. In sum, this analysis shows that inefficient conglomerates can persist due to division managers' rent-seeking activities that prevent headquarters from breaking up the conglomerate.

VII. Redeployable Capital

Unlike a single-division firm, a conglomerate can internally reallocate at relatively low cost idle or underutilized capital in one division to another division where productivity may be higher. In this section, we analyze the effect of capital redeployment on divisional investment, payout, liquidation, refinancing decisions, and the conglomerate's value.

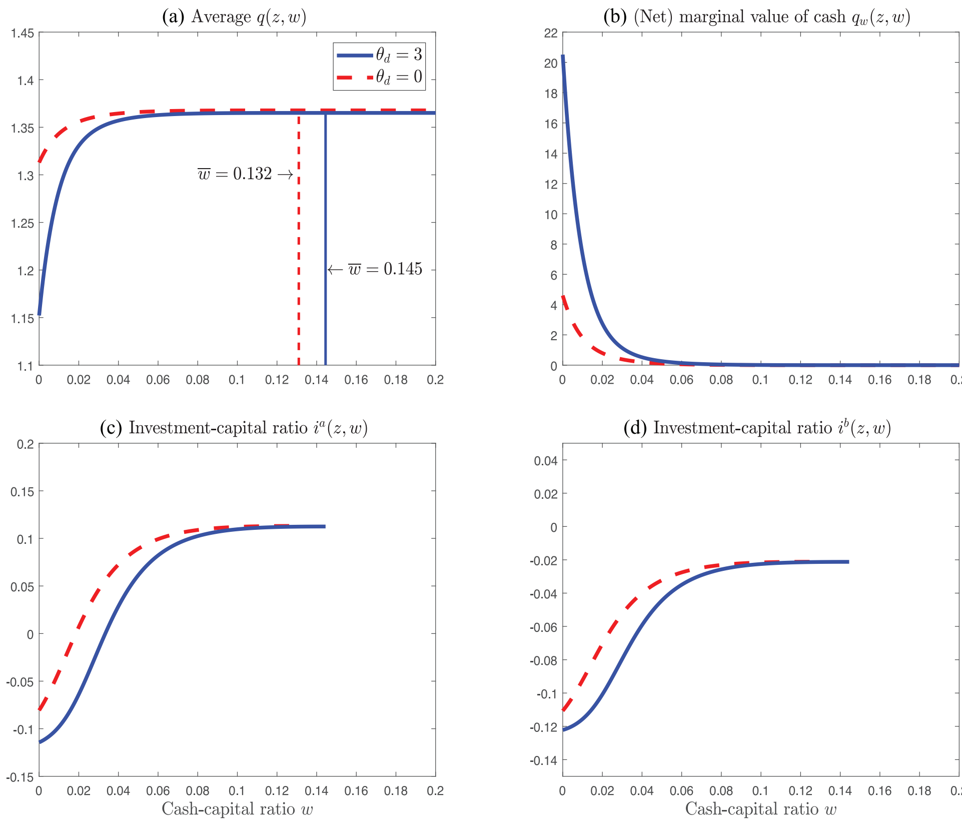

Figure 10. Comparison of diversified firms with ($\theta_d = 3$) and without ($\theta_d = 0$) socialism costs with respect to the spinoff decision in the liquidation case. Both firms are subject to socialism costs with respect to their ongoing operations ($\theta_c = 1$). The firms have asymmetric divisions with $\mu_a = 24\%$ and $\mu_b = 16\%$. The divisions are of equal size ($z = 0.5$). In Panel A, the vertical lines mark the payout boundary $\overline{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

A. Model and Solution

We model capital redeployability by generalizing the capital adjustment cost function. In our baseline model, capital adjustment costs are additively separable at the division level and hence by assumption there is no capital redeployment between the two divisions. In practice, however, redeploying capital within the firm can be cheaper than acquiring or selling capital goods in the marketplace. To capture both components of the adjustment costs, we assume that the conglomerate's total capital adjustment cost at t, G_t, is given by

$$G_t = (1 - \chi)(G_t^a + G_t^b) + \chi G_t^{re}, \tag{60}$$

where $\chi \in [0, 1]$ is a constant, G^a and G^b are the capital adjustment costs for divisions a and b, respectively, given in (4), and G^{re} captures the adjustment costs of redeploying capital from one division to the other within the firm. We again choose a homogeneous adjustment cost function by specifying $G_t^{re} = G^{re}(I_t, K_t) = g_{re}(i_t)K_t$, where $I_t = I_t^a + I_t^b$ is the firm's investment, $K_t = K_t^a + K_t^b$ is the firm's total capital stock, $i_t = I_t/K_t = z_t i_t^a + (1 - z_t)i_t^b$ is the firm's investment-to-capital ratio, and $g_{re}(i_t)$ is the scaled adjustment cost for capital redeployment.

The parameter χ measures the contribution of the firm-level capital redeployment cost to the total adjustment cost G_t. When $\chi = 0$, we uncover our baseline formulation in which there is no capital redeployment between the two divisions. When $\chi = 1$, the firm incurs capital redeployment costs only at the firm level and there are no division-specific adjustment costs. The q-theoretic models in the literature typically specify adjustment costs at the firm level, which corresponds to the $\chi = 1$ case. Empirically, adjustments tend to be smoother at the firm level than at the plant level (e.g., Doms and Dunne (1998)). Our more general adjustment cost specification (60) captures this empirical feature.

Next, to exposit the economic mechanism of capital redeployment, consider the special (symmetric) case, where the (scaled) adjustment costs are the same at the divisional level (for both divisions a and b) and the conglomerate level:

$$g_a(\cdot) = g_b(\cdot) = g_{re}(\cdot) \equiv g(\cdot). \tag{61}$$

We show that being able to redeploy capital across divisions makes the firm more cost effective by lowering its total adjustment costs, in that

$$G_t^{re} \le G_t^a + G_t^b, \tag{62}$$

which follows from Jensen's inequality.[44] The intuition for this result is as follows. Because adjustment costs are convex, paying the cost once at the firm level via the capital redeployment channel is cheaper than paying the adjustment cost twice at the divisional level (once for each division). Moreover, the higher the value of χ, the more the capital redeployment between the two divisions contributes to the firm's total adjustment costs, and the lower the conglomerate's total adjustment costs.

Next, we turn to the firm's investment policies. The FOCs with respect to investment at each of the firm's divisions now depend on the adjustment cost functions at both the divisional and the firm level. Consider investment of division s, where $s = a, b$. With capital redeployment ($\chi \in (0, 1]$), the marginal cost of investing in K^s is

$$1 + (1-\chi)G^s_{I^s}(I^s, K^s) + \chi G^{re}_{I^s}(I^a + I^b, K^a + K^b) = 1 + g'(i^s) - \chi(g'(i^s) - g'(i)). \tag{63}$$

[44] Using the homogeneity property, $i_t = z_t i_t^a + (1 - z_t)i_t^b$, the simplifying assumption (61), and the convexity of $g(\cdot)$, we obtain (62) from

$$\frac{G_t^a + G_t^b}{K_t^a + K_t^b} = \frac{g_a(i_t^a)K_t^a + g_b(i_t^b)K_t^b}{K_t^a + K_t^b} = g(i_t^a)z_t + g(i_t^b)(1 - z_t) \ge g(i_t) = \frac{G_t^{re}}{K_t^a + K_t^b}.$$

Compared to our baseline model ($\chi = 0$), where the firm's marginal cost of investing in K^s is $1 + G^s_{I^s}(I^s, K^s) = 1 + g'(i^s)$, the last term in (63) is new and captures the effect of redeployment on the FOC for I^s. When the investment-to-capital ratio of division s is higher than that of the other division, we have $i^s > i$, which implies $g'(i^s) - g'(i) > 0$ (convexity), effectively reducing the marginal cost of investing in K^a, as can be seen from the right-hand side of (63). The FOC for $\{I^s,\ s = a, b\}$ can then be written as

$$1 + g'(i^s) - \chi(g'(i^s) - g'(i)) = \frac{F_{K^s}(K^a, K^b, W)}{F_W(K^a, K^b, W)}, \tag{64}$$

where the right-hand side corresponds to the endogenous marginal benefit of investing, given by the ratio of the marginal q of division s, $F_{K^s}(K^a, K^b, W)$, divided by the firm's marginal value of cash $F_W(K^a, K^b, W)$, as in our baseline model. Compared to our baseline model, the new term is $-\chi(g'(i^s) - g'(i))$ on the left-hand side of (64) that captures capital redeployability. A key takeaway from FOC (64) is as follows. For a given (endogenous) marginal benefit of investing (on the right-hand side), the division whose investment-to-capital ratio is larger than the other invests more with capital redeployment than without, which can be seen from the convexity of $g(\cdot)$. Because the more productive division (division a) invests more on average than the other division (and hence $g'(i^a) > g'(i)$), we conclude that capital redeployment effectively boosts the more productive division a's productivity. Section VII.A of the Internet Appendix provides the model solution.

B. Quantitative Analysis

Next, we use numerical solutions to illustrate the effect of capital redeployment for the liquidation case.[45] We consider the case with asymmetric productivities ($\mu_a = 24\%$ and $\mu_b = 16\%$) and equal-sized divisions ($z = 0.5$). We compare the case in which the firm has an option to redeploy capital across divisions ($\chi = 0.5$) to our baseline case without this option ($\chi = 0$). We set $\chi = 0.5$ so that the cost associated with capital redeployment at the firm level, $g(i)$, is in between our baseline case ($\chi = 0$) and the other extreme case ($\chi = 1$), where capital adjustment costs are only paid at the firm level. We set $\theta_a = \theta_b = \theta_{re} = 8$ in the (scaled) quadratic adjustment cost functions at both the divisional and the firm level.

Figure 11 plots the solution. Panel A shows that the option to redeploy capital ($\chi = 0.5$) within the conglomerate increases the conglomerate's average q. Moreover, the capital redeployment option is more valuable when cash is abundant (high w). This result is intuitive, as conglomerates that are less financially constrained are better able to achieve the efficiency gains from redeployability. Panel B further shows that the marginal value of cash is higher with capital redeployability. This result is intuitive as well. Effectively, capital

[45] Section VII.B of the Internet Appendix analyzes the refinancing case.

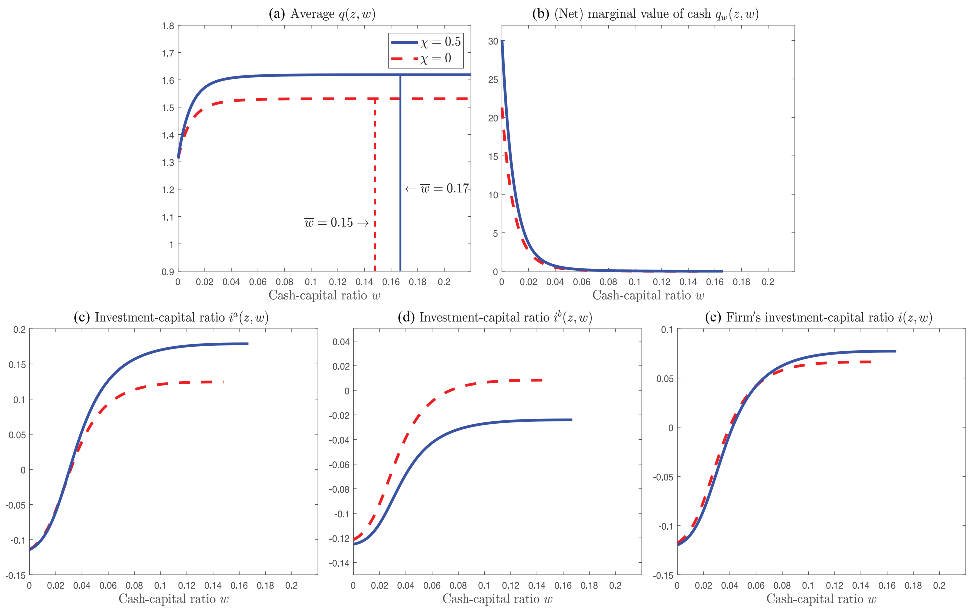

Figure 11. Comparison of diversified firms with ($\chi = 0.5$) **and without** ($\chi = 0$) **redeployable capital in the liquidation case**. The firms have asymmetric divisions with $\mu_a = 24\%$ and $\mu_b = 16\%$. The divisions are of equal size ($z = 0.5$). We set $\theta_a = \theta_b = \theta_{re} = 8$ in the (scaled) quadratic adjustment cost functions at both the divisional and the firm level. In Panel A, the vertical lines mark the payout boundary $\overline{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

redeployability makes the conglomerate more efficient, thereby increasing the marginal value of cash.

Panels C and D show that the option to redeploy capital makes the more productive division a invest more (for sufficiently large values of w), and the less productive division b invest less. That is, with capital redeployability, the conglomerate allocates physical capital more efficiently between the two divisions. The net effect, shown in Panel E, is that the firm's investment i is higher with capital redeployability than without (for sufficiently large values of w). Our results further show that asset sale (i.e., negative investment) is an efficient way to manage risk when w is sufficiently low, as avoiding inefficient liquidation is more valuable with capital redeployability than without. This explains why the two lines in Panels C and E cross (although they do not substantially depart from one another for low values of w).

C. Redeployable Capital with Socialism

In Section VII.C of the Internet Appendix, we consider an extension of our baseline model that features both redeployable capital and corporate socialism. An interesting insight from this analysis is that capital redeployability mitigates the resource misallocation under socialism. Intuitively, capital redeployability provides a "hedge" against corporate socialism, as it reduces the cost of allocating resources in a way that decreases the socialism costs.

VIII. Extensions

In the Internet Appendix, we present three additional extensions of our baseline model, which we briefly summarize in this section.

A. Alternative Specification of Spinoff Payoffs

In Section VIII of the Internet Appendix, we extend our baseline model by allowing the conglomerate to optimally allocate a fraction of its cash holdings to the division that it plans to spin off. Doing so alleviates the financial constraints of the sold division, which increases the price that potential buyers are willing to pay for the division. This alternative specification of the spinoff payoff is natural in settings in which the sold division becomes a stand-alone firm held by well-diversified financial investors that value the firm as a going-concern entity.

In the quantitative analysis, we show that, in contrast to our baseline model, early liquidation can be optimal even with symmetric divisions. This is because, by optimally allocating a fraction of its cash to the to-be-sold division, the firm (with the remaining division) fetches the highest value for its shareholders. Liquidating the division upon exhausting its cash is suboptimal as the selling price of the liquidated division would be too low.

B. Endogenous Formation of the Conglomerate

In Section IX of the Internet Appendix, we generalize our baseline framework by also modeling the initial transition from a stand-alone firm to a conglomerate. That is, the firm starts as a stand-alone firm and considers other stand-alone firms as potential targets for an acquisition. Upon completing the acquisition, the firm becomes the conglomerate of Section I.

This generalized framework allows us to characterize the endogenous formation of conglomerates. In particular, our analysis shows that, when the single-division firm is flush with cash, the decision to become a conglomerate is similar to an investment decision that helps achieve diversification benefits. In contrast, when the single-division firm runs out of cash, the decision is closer to a financing decision that helps replenish the firm's liquidity when the firm faces high external financing costs.

C. Conglomerate Premium and Discount

In Section X of the Internet Appendix, we extend our generalized model from Section IX of the Internet Appendix to study how the endogenous formation of the conglomerate can lead to either a conglomerate discount or premium.[46]

In that extension, we allow managers to derive private benefits from building a conglomerate (e.g., in the form of empire-building preferences). Because of these private benefits, the manager is willing to overpay for the target, which can outweigh the diversification benefits and lead to a conglomerate discount. Taking these forces into account, our analysis shows that agency frictions have a nonlinear and nonmonotonic impact on the diversification premium and discount. In this regard, our results echo the mixed findings from the empirical literature and highlight the importance of considering the endogenous formation of the conglomerate in empirical studies of the conglomerate discount/premium.

IX. Conclusion

In this paper, we provide a tractable model in which investment, cross-divisional transfers, division sale (spinoff), cash management, external financing, and dividend payout are jointly characterized for a multidivision firm that faces costly external finance. Our model provides a rich set of novel predictions, ranging from a refined formulation of the winner-picking role of internal capital markets to a characterization of the optimal spinoff decision. Moreover, we develop a q theory of investment for *financially constrained* multidivision firms, in which division-level investment is determined by the ratio between the marginal q for that division's capital stock and the marginal value of cash of the multidivision firm. Finally, we consider several extensions of our baseline model. In particular, we allow for capital redeployability and account for the endogenous formation of the conglomerate.

While our model allows for rent-seeking behavior of the division managers, it does not speak to the optimal contract design. In this regard, enriching our model with a dynamic contracting framework-for example, of the type studied by Malenko (2019) for capital budgeting—could be a fruitful extension. Doing so would provide a characterization of the optimal contract that arises taking into account the complexity and intertwined nature of the multidivision firm's policies.

[46] A large empirical literature studies the conglomerate discount. This literature started with the work by Lang and Stulz (1994) and Berger and Ofek (1995), who find that conglomerates were valued at a discount compared to synthetic portfolios of single-segment firms that match the conglomerates' composition. This finding was challenged by subsequent literature. In particular, Campa and Kedia (2002), Graham, Lemmon, and Wolf (2002), and Villalonga (2004) show that the discount can be explained by the self-selection of firms that choose to become conglomerates. After accounting for the endogenous formation of the conglomerate, they find only mixed support for the conglomerate discount, which sometimes turns into a conglomerate premium. Our generalized framework is helpful in this context, as it explicitly models the endogenous formation of conglomerates, shedding light on the forces that can generate a conglomerate discount and premium, respectively. For a review of the empirical literature on the conglomerate discount, see Maksimovic and Phillips (2013).

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher’s website:

Appendix S1: Internet Appendix.
Replication Code.